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Competition in the Financial Sector and Financial Crises in a Business Cycle Model

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Abstract

This theoretical work studies a dynamic general equilibrium model with the financial sector in which aggregate activity depends on the conditions of intermediaries' balance sheets. This environment is used to demonstrate the business cycle consequences of changes in competition in the financial industry. On the one hand, a more competitive banking sector is associated with a higher average level of aggregate output. On the other hand, however, a less competitive financial industry increases financial and macroeconomic stability. This trade-off is present both in the short run and in the long run.

Keywords: financial crisis, banking competition

JEL Classification: D43, E44, G21

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1 Introduction

The goal of this paper is to investigate questions that arose during the Global Financial Crisis of 2007–2009: is financial stability enhanced or weakened by the competition in the financial industry? What are the business cycle implications of changes in the financial sector's market structure?

A significant increase in competition in the US financial sector started in early 1970's. It was additionally strengthened by two policy reforms: the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Garn-St. Germain Depository Institutions Act of 1982. Conventional wisdom linking this deregulation of financial markets with the financial instability and the Global Financial Crisis of 2007-2009 was expressed in the article "Deliver us from competition" published in The Economist in 2009: *"The severity of today's financial crisis is blamed by some on the pressure of competition on banks. (...) the lifting of restraints, such as interest-rate caps on deposits or rules that prevent banks from operating in certain markets, leads to more intense competition. That is good for borrowers, but it also hurts banks' profit margins".* To put it differently, the rise in competition led to lower margins and lower profits, which made it harder for banks to collect equity. This, in turn, resulted in the financial instability and was one of the causes of the financial crisis.

Motivated by this example, this article studies the impact of changes in the competition in the financial sector on financial stability and business cycle performance of the economy. To conduct my analysis, a tractable business cycle model is constructed that has several features that make it useful for studying this topic. First, it includes the financial sector (also referred to as banks) transferring funds from agents who do not have investment opportunities to those who have them. Second, the level of aggregate investment is determined by the conditions of banks' balance sheets. In particular, if the equity of financial institutions is drained by an adverse aggregate shock then the intermediation activities are impeded. This, in turn, means that fewer resources are transferred to investors and hence the level of aggregate investment drops. This lowers the stock of physical capital and output. Third, intermediaries provide depositors with safe assets with a return that does not depend on the realization of aggregate shocks. This feature implies that financial intermediaries in my model are similar to standard banks as they provide agents with services resembling deposits. Fourth, my model allows for the comparison of the financial market structures characterized by different intensities of monopoly.

The main result can be viewed as a short-run trade-off between two economies: the one with competitive and the other with monopolistically competitive banks. In particular, those results focus on two opposite forces: on the one hand, competitive banks channel more funds to the investors leading to higher production of investment goods in the economy. This, in turn, increases the accumulation of physical capital and boosts aggregate output. On the other hand, however, competitive bankers exhibit greater risk exposure and hence incur more severe losses during recessions which

drains their equity. This impedes the intermediation activities and therefore lowers investment and output in the future.
In Appendix A, it is shown that an analogous trade-off is present in the long run, too. More precisely, it concentrates on the properties of ergodic distributions of two aggregate state variables (aggregate capital and banks' equity) under two financial regimes (competitive vs. monopolistic banks). Analogously to the short run, a trade-off is observed: on average, competitive banks provide entrepreneurs with a larger amount of cheaper intermediation services. At the same time, lower profit margins generated by more competitive banks hinder the accumulation of equity. This, in turn, deteriorates financial (and macroeconomic) stability.
The rest of the paper is organized as follows. Section 2 discusses the related literature and presents the contributions of my work. Section 3 presents the business cycle model with perfectly competitive banks which is used for the analysis of the transmission mechanism of aggregate shocks. Section 4 describes the model with monopolistically competitive intermediaries nesting the model competitive banks as a special case. In Section 5, a comparison of the two regimes: the economy with perfectly competitive financial institutions and the economy with monopolistically competitive banks is made. Section 6 concludes.

2 Literature

The paper is related to several strands of the literature.

Market structure and financial stability. The first, theoretical strand, concerns the effects of changes to the banking sector's market structure on its stability. There are two main approaches in this literature: the risk-shifting view and the charter value view. The risk-shifting theory, represented by Boyd and Nicolo (2005) (that builds on the seminal work by Stiglitz and Weiss (1981)), argues that an increase in the monopoly power in the banking sector leads to higher interest rates on bank loans which makes firms invest in riskier projects. This, in turn, translates into higher banks' portfolio risk and gives rise to financial instability. By contrast, the charter value hypothesis, originating from Keeley (1990), postulates that a decrease in the competition in the banking industry increases banks' future profits making them more cautious when making investment decisions. This happens because risky behavior may cause a bankruptcy meaning that they lose a valuable stream of future rents.
Martinez-Miera and Repullo (2010) try to reconcile the two aforementioned views. They claim that on the one hand when (as a result of decrease in intermediaries' monopoly power) banks charge lower rates, their borrowers choose safer investments, so their portfolios are safer (as in Boyd and Nicolo (2005)). On the other hand, lower interest rates on loans decrease banks' profits which serve as a buffer against loan

losses. Those two opposite forces give rise to an U-shaped relationship between the monopoly power and the risk of bank failure.
My analysis intermediaries operate under an implicit no-default constraint and hence there are no bank failures. This does not mean, however, that the issue of financial stability does not emerge: this is because the amounts of intermediation and aggregate investment depend on banks' equity in my model. Specifically, if the aggregate level of banks' equity is low then so is the resource reallocation through the financial sector and aggregate investment. As a consequence, financial shocks that drain banks' equity lead to lower aggregate investment and recessions. If financial intermediaries' have monopoly power then they are able to accumulate an equity cushion that buffers them against potential financial shocks. So the first part of the "trade-off" in my model is similar to the force described by Martinez-Miera and Repullo (2010). The second part, however, has nothing to do with the investment risk choice made by firms as in Stiglitz and Weiss (1981). It is rather a standard textbook mechanism that makes monopolistic banks less favorable: monopolistic intermediaries channel less resources and they impose higher spreads than competitive bankers. As a result, in normal times the level of aggregate investment is lower which in turn decreases aggregate capital stock and output.
To my best knowledge, there are no papers that describe the impact of financial intermediaries' market structure on the real economy in the context of business cycle fluctuations. This work is intended to fill this gap by incorporating a simple banking system into otherwise standard neoclassical framework. Additionally, the analysis captures both the dynamic and general equilibrium effects that were ignored in some articles cited above that have a static or a partial-equilibrium character.

Dynamic equilibrium models. There is an immense literature on financial frictions and the role of the banking sector in the RBC framework. There are, however, two works closely related to this paper using a similar formalization techniques to address the role of banks in the economy.
Firstly, my model builds on Kiyotaki and Moore (2019). To give rise to trade in capital, Kiyotaki and Moore (2019) split the population of entrepreneurs into two segments: investors (that hold investment opportunities) and those who do not have them. Investors issue equity claims (that entitle their holders to capital income streams) to finance their projects which are purchased by those who cannot invest. This division of the population gives rise to trade in assets. A similar construction is used to generate endogenous reallocation of capital in the analyzed model. There is, however, a fundamental difference between Kiyotaki and Moore (2019) and my work. In Kiyotaki and Moore (2019), agents do not need services provided by intermediaries to sell/purchase capital whereas in my model only banks can channel capital between the two types of entrepreneurs.
From the technical point of view, the closest article to mine is Bigio and d'Avernas (2021). Similarities between my work and Bigio and d'Avernas (2021) entail: the

presence of the two types of entrepreneurs (consumption goods and investment goods producers) and banks that transfer capital sold by investment goods producers to consumption goods producers.There is, however, a significant difference: unlike Bigio and d'Avernas (2021), my model does not include the asymmetric information about capital quality. Instead, to generate a strictly increasing supply of capital in the model, it is assumed that investment goods producers have different productivity levels and hence some of them are more willing to sell their capital than the others to be able to undertake investment projects.
Presented analysis focuses on the consequences of the banks' balance sheet's conditions for the dynamics of aggregate output and investment. As such, it abstracts from other fundamental questions like the coexistence of money and credit and how financial intermediariation affects the allocation when both assets are in place. Those topics were covered in important papers by by Berentsen et al. (2007) and Gu et al. (2016). In particular, Berentsen et al. (2007) introduce financial intermediation into a monetary model based on the seminal paper by Lagos and Wright (2005). They find that in the economy where agents are subject to trading shocks there is a substantial role for financial intermediaries that accept nominal deposits and make nominal loans. Berentsen et al. (2007) find that intermediation improves the allocation and the related welfare gains come from the payment of interest on deposits and not from relaxing borrowers' liquidity constraints.

3 Economy with perfectly competitive intermediaries

3.1 Environment

Time. Time is infinite and divided into discrete periods. Each period consists of two subsequent stages.

Agents. The model is populated by three classes of agents: infinitely-lived entrepreneurs (also called producers), infinitely-lived financial intermediaries (called banks, too) and workers. First two populations have measures normalized to one. Population of workers has measure L. Financial intermediaries are identical and there are two types of entrepreneurs: consumption goods producers and investment goods producers that have measures π_C and $\pi_I = 1 - \pi_C$, respectively. In what follows, they are referred to as c-producers/c-entrepreneurs and i-producers/i-entrepreneurs, too.

Shocks. There is one aggregate shock: an i.i.d. shock $Z_t \in \mathbb{R}_+$. It affects the demand (of c-producers) for the capital transferred by intermediaries and gives rise to the portfolio risk faced by banks. Moreover, there is an idiosyncratic uncertainty faced by entrepreneurs: at the beginning of the first stage, entrepreneurs are randomly segmented into two subgroups: c-producers and i-producers. This division generates two separate populations of entrepreneurs: those who consider selling their capital to finance their investment projects (i-producers) and those who want to purchase capital (c-entrepreneurs). Additionally, each i-entrepreneur draws the productivity level that is associated with his investment opportunity which is an additional source of idiosyncratic uncertainty faced by producers. It will be clear later that introducing investment opportunities featuring different productivity levels gives rise to a differentiable and monotonically increasing supply of capital. By contrast to the i-entrepreneurs, all c-producers operate identical production technology.

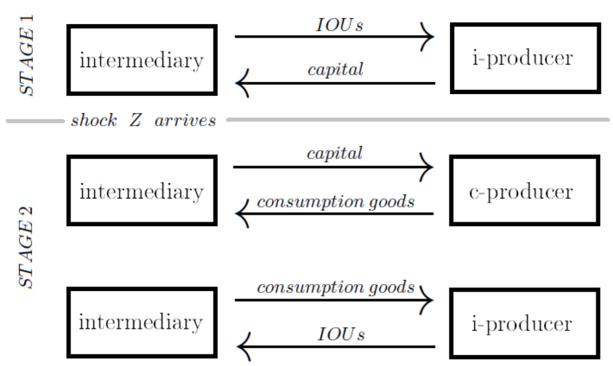
Goods, technologies and trade. There are two types of goods: capital goods and consumption goods, and two production factors: capital and labor. C-entrepreneurs use their capital holdings k and hire l workers (that are paid wage w) to produce consumption goods. They operate the Cobb-Douglas technology $A_C k^{\alpha} l^{1-\alpha}$ where A_C is technology level that is equal across c-entrepreneurs. C-producers are not able to manufacture capital goods. Since their capital holdings depreciate (this occurs between periods at rate δ), they are willing to increase it and hence they have incentives to purchase capital.

Consumption goods can be transformed into capital by i-entrepreneurs. They have an access to a linear technology that generates $A_I i$ capital goods (that increase the i-producers capital holdings in the next period) out of i consumption goods. It is assumed that A_I varies across i-producers. In particular, A_I is drawn from the probability distribution described by a continuous density function $f(A_I)$ that satisfies $\mathbb{E}A_I < +\infty$ and $\text{supp}(f) = \mathbb{R}_+$. It is assumed that f is continuous because it guarantees that the aggregate supply of capital is a differentiable function. $\mathbb{P}_{A_I}$ denotes the probability measure associated with A_I. The amount of consumption goods used to generate capital is called investment. I-entrepreneurs are unable, however, to use their capital holdings to produce consumption goods. To get them, they have to sell their capital holdings.

Workers are identical. Each worker supplies inelastically one unit of labor and it is assumed that they do not have access to financial markets so they simply consume their wages each period. Workers are introduced to the model to guarantee that c-producers' profits are linear in capital holdings. This enables me to derive the analytical formulas for the c-entrepreneurs' policies and to ensure that my environment is stationary (since the production technology of consumption goods is concave in capital given the Cobb-Douglas technology).

C-entrepreneurs and i-entrepreneurs cannot trade capital and consumption goods directly, they have to use services provided by banks. This assumption, similar to the one made by Bigio and d'Avernas (2021), mechanically gives rise to a crucial role of financial intermediaries. In equilibrium, during the first stage, intermediaries buy capital from i-producers (capital sellers) at price q_S and they finance their purchases with riskless IOUs that they issue. At the same time, c-producers generate consumption goods. At the end of the first stage value of aggregate shock Z is realized. During the second stage, banks transfer capital to capital buyers. Intermediary gets q_B consumption goods produced by capital purchasers (c-entrepreneurs) for one unit of capital sold to them. At the end of the second stage, banker transfers consumption goods to capital sellers to settle their debt (IOUs). All agents consume at the end of the second stage and i-entrepreneurs produce capital using consumption goods received from banks as an input. The sequence of transactions is presented in Figure 1. It is assumed that intermediaries cannot default on their debt (i.e., IOUs held by capital sellers) and that they are not able to store capital after the arrival of the shock (if this assumption is relaxed then the portfolio risk faced by banks is eliminated because, in the case of an adverse economic shock, banks could store capital and wait for better market conditions to sale it at higher prices). The latter implies that they transfer the total amount of capital purchased from i-entrepreneurs. On the other hand, banks have the technology to store consumption goods between periods so they are able to accumulate equity over time (which means that from the technical point of view it is a stock of consumption goods). By contrast, the only storage technology available to producers is the capital storage technology. Notice that the no-default constraint has an implication for the character of the contract between capital sellers (i-entrepreneurs) and intermediaries: it resembles a standard deposit because it does not depend on changes in the aggregate conditions. Summing up, the assumptions related to the financial intermediation in the model are necessary to generate the portfolio risk faced by banks and the provision of riskless assets (deposits) - distinctive features of a realistic banking sector.

Figure 1: Financial intermediation



Notes: The figure shows the timeline of transactions within the period. More details on timing is provided in Subsection 3.1 (paragraph Goods, technologies and trade).

Preferences. Workers, bankers and i-entrepreneurs have preferences over lifetime consumption streams $\{c_t\}_{t=0}^{+\infty}$ described by:

$$\mathbb{E}_0\left(\sum_{t=0}^{+\infty}\beta^t u(c_t)\right),$$

where u is a strictly increasing and strictly concave function of c_t and $0<\beta<1$ is their discount factor. Observe, that it is common for the models in the literature to assume linear preferences of intermediaries. However, the concave intermediary's utility function u can be justified by dividend-smoothing motives (applied to entrepreneurs by Jermann and Quadrini (2012)). Additionally, a recent use of concave preferences of bankers can be found in Brunnermeier and Sannikov (2014). This assumption is made because it guarantees the existence of the interior solution to the banker's problem and hence it enables comparative statics exercises. Moreover, it engenders a slow recovery dynamics after the crisis.
C-producers have preferences that depend on the aggregate shock Z_t:

$$\mathbb{E}_0\left(\sum_{t=0}^{+\infty}\beta^t \cdot Z_t \cdot u(c_t)\right).$$

This dependence is introduced to give rise to shifts in the demand for capital purchased from intermediaries. If Z_t is high then c-entrepreneurs value consumption more and their demand for capital drops. Assumptions made in this section are discussed in a more detailed way in Appendix A.

3.2 Optimization problems

Workers. As mentioned, workers are hand-to-mouth. This means that they simply consume their wages w_t:

$$c_t = w_t. \tag{1}$$

I-producers. Let us start with the dynamic problem of the i-producer that begins period with capital holdings k and is affected by productivity shock A_I. From the description of the intermediation process, we know that it makes decisions in the first stage. The corresponding Bellman equation reads:

$$V^I(k,K,E,A_I) = \max_{c>0,i\geq 0,k_S>0,k'>0}\Big\{\log(c) + \beta\mathbb{E}_{Z,Z',A_I'}\left(\pi_I\cdot V^I(k',K',E',A_I')\right.$$
$$\left.+\ \pi_C\cdot V^C(k',K',E',Z')|K,E\right)\Big\}$$

subject to:

$$\begin{cases} c + i = q_S(K,E) \cdot k_S, \\ k' = A_I \cdot i + (1-\delta)(k - k_S), \\ E' = E'(K,E,Z), \\ K' = K'(K,E), \end{cases} \tag{2}$$

where V^I is the value function associated with the dynamic maximization problem of i-entrepreneur and V^C is the value function associated with the problem of c-producer. The prime symbols denote next period values. Observe that arguments of V^I and V^C are different: it is because i-entrepreneurs make decisions (about selling capital) in the first stage, before the realization of Z and because c-producers do not face idiosyncratic uncertainty associated with their productivity levels. E denotes the aggregate stock of banks' equity. The first equation that determines the set of possible actions is the budget constraint of the i-entrepreneur: it says that i-producer sells k_S units of its capital holdings at price $q_S(K,E)$ and uses the proceedings (consumption goods) for investment i and consumption c. Second constraint is the law of motion for individual capital holdings. Observe that the amount of capital generated out of i consumption goods depends on the productivity level A_I. Expression $(1-\delta)(k-k_S)$ denotes the unsold capital that depreciates at rate δ. Third and fourth constraints describe the perceived laws of motion for aggregate banks' equity E and aggregate capital K (i.e., it captures an implicit assumption about agents' rational expectations).

Notice that the logarithmic form of utility is assumed. It will be shown that this assumption guarantees that entrepreneurs' (intermediaries') policy functions are linear in capital holdings k (or bank's equity holdings e). This coupled with the assumption about the capital holdings' independence of productivity shocks means that the distribution of entrepreneurs' capital holdings is not a state variable, which greatly simplifies the aggregation exercise.

Observe that if the i-entrepreneur's productivity A_I is sufficiently high then he may decide to sell all his capital holdings k to finance his investment i (and consumption). On the other hand, if it is low enough, then i-producer decides to reduce the amount of capital that is sold and sets $i = 0$. The following lemma formalizes this intuition:

Lemma 1. *Suppose that i and k_S solve (2). If $A_I \geq A_I^*(q_S)$ then $i > 0$ and $k_S = k$. If $A_I < A_I^*(q_S)$ then $i = 0$ and $0 < k_S < k$. The value $A_I^*(q_S)$ satisfies:*

$$A_I^*(q_S) = \frac{1-\delta}{q_S(K,E)}.$$

All the proofs are delegated to Appendix B (the proofs related to the long-run analysis, presented in Appendix A, are delegated to Appendix C.).

Lemma 1 is useful as it allows for splitting the i-producer's problem (2) into two separate problems admitting interior solutions.

The first problem pertains to an i-entrepreneur with productivity A_I satisfying $A_I \geq A_I^*(q_S)$:

$$V^{I_P}(k,K,E,A_I) = \max_{c>0, i\geq 0, k'>0} \Big\{ \log(c) + \beta \mathbb{E}_{Z,Z',A_I'} \left(\pi_I \cdot \mathbb{P}_{A_I}\left(A_I \geq A_I^*(q_S')\right) \times \right. \\ \times\ V^{I_P}(k',K',E',A_I') + \pi_I \cdot \mathbb{P}_{A_I}\left(A_I < A_I^*(q_S')\right) \cdot V^{I_0}(k',K',E') + \\ \left. + \ \pi_C \cdot V^C(k',K',E',Z') | K,E \right) \Big\}.$$

subject to:

$$\begin{cases} c + i = q_S(K,E) \cdot k, \\ k' = A_I \cdot i, \\ E' = E'(K,E,Z), \\ K' = K'(K,E), \end{cases}$$

where V^{I_P} is value function associated with the problem of an i-entrepreneur with the current productivity level $A_I \geq A_I^*(q_S)$ that produces new capital, V^{I_0} is value function that corresponds to the problem of i-producer with a relatively low productivity (i.e., $A_I < A_I^*(q_S)$) and it sets its investment at the level $i = 0$. Budget constraint indicates that i-entrepreneur with $A_I \geq A_I^*(q_S)$ sells his entire capital k and the law of motion for his capital shows that his future capital holdings come entirely from the creation of new capital.

The second problem corresponds to i-producer that has low productivity: $A_I < A_I^*(q_S)$. Budget constraint shows that $i = 0$ and entrepreneur does not sell his entire capital holdings as $k' > 0$. According to the law of motion, the unsold capital depreciates and becomes producer's capital holdings in the next period:

$$V^{I_0}(k,K,E) = \max_{c>0, k_S>0, k'>0} \Big\{ \log(c) + \beta \mathbb{E}_{Z,Z',A_I'} \left(\pi_I \cdot \mathbb{P}_{A_I}\left(A_I \geq A_I^*(q_S')\right) \times \right. \\ \times\ V^{I_P}(k',K',E',A_I') + \pi_I \cdot \mathbb{P}_{A_I}\left(A_I < A_I^*(q_S')\right) V^{I_0}(k',K',E') \ + \\ \left. + \ \pi_C \cdot V^C(k',K',E',Z') | K,E \right) \Big\}.$$

subject to:

$$\begin{cases} c = q_S(K,E) \cdot k_S, \\ k' = (1-\delta)\left[k - k_S\right], \\ E' = E'(K,E,Z), \\ K' = K'(K,E). \end{cases}$$

C-producers. This group of entrepreneurs makes decisions in the second stage, after the realization of aggregate shock Z. They choose their consumption, capital purchases and the number of workers hired:

$$V^C(k,K,E,Z) = \max_{c>0,k_B\in\mathbb{R},k'>0,l>0} \Big\{ Z\cdot\log(c) + \beta\mathbb{E}_{Z',A_I'}\left(\pi_I\cdot\mathbb{P}_{A_I}\left(A_I'\geq A_I^*(q_S')\right)\times\right.$$
$$\times\ V^{I_P}(k',K',E',A_I') + \pi_I\cdot\mathbb{P}_{A_I}\left(A_I'<A_I^*(q_S')\right)V^{I_0}(k',K',E') +$$
$$\left. +\ \pi_C\cdot V^C(k',K',E',Z')|K,E\right)\Big\}.$$

subject to:

$$\begin{cases} c + q_B(K,E,Z)k_B = A_C k^\alpha l^{1-\alpha} - w(K)\cdot l, \\ k' = (1-\delta)\left[k+k_B\right], \\ E' = E'(K,E,Z), \\ K' = K'(K,E), \end{cases} \tag{3}$$

where $q_B(K,E,Z)$ is the price at which c-entrepreneurs buy assets from intermediaries and k_B is the amount of purchased capital. We will see that in equilibrium $k_B > 0$. Observe that Z affects the c-producer's preferences giving rise to changes in demand for asset purchases k_B.

Since l enters only the RHS of c-producer's budget constraint, problem (3) can be analyzed in two stages: first, c-producer's profits $A_C k^\alpha l^{1-\alpha} - w(K)\cdot l$ are maximized with respect to l and then the maximization problem is solved with respect to the remaining variables: $c > 0$, $k_B \in \mathbb{R}$, $k' > 0$. The value of l that solves the first maximization problem satisfies:

$$l^* = \left(\frac{(1-\alpha)A_C}{w(K)}\right)^{1/\alpha}\cdot k. \tag{4}$$

Plugging solution l^* into the dynamic problem (3) yields:

$$V^C(k,K,E,Z) = \max_{c>0,k_B\in\mathbb{R},k'>0} \Big\{ Z\cdot\log(c) + \beta\mathbb{E}_{Z',A_I'}\left(\pi_I\cdot\mathbb{P}_{A_I}\left(A_I'\geq A_I^*(q_S')\right)\times\right.$$
$$\times\ V^{I_P}(k',K',E',A_I') + \pi_I\cdot\mathbb{P}_{A_I}\left(A_I'<A_I^*(q_S')\right)V^{I_0}(k',K',E') +$$
$$\left. +\ \pi_C\cdot V^C(k',K',E',Z')|K,E\right)\Big\}.$$

subject to:

$$\begin{cases} c + q_B(K,E,Z)k_B = G(K)\cdot k, \\ k' = (1-\delta)\left[k+k_B\right], \\ E' = E'(K,E,Z), \\ K' = K'(K,E), \end{cases}$$

where $G(K)$ satisfies $G_K < 0$ and is given by:

$$G(K) = \alpha A_C\left(\pi_C\cdot[K/L]\right)^{\alpha-1}.$$

This means that the RHS of the c-entrepreneur's budget constraint is linear in his capital holdings k. This property is useful in the next subsection in which characterizes policy rules and value functions of entrepreneurs.

Characterization of decision rules. It will be shown that given the logarithmic preferences and budget constraints linear in asset holdings, policy functions associated with the maximization problems listed above are linear in producer's capital holdings k. This enables to aggregate the decisions made by all producers within each segment (i-entrepreneurs and c-entrepreneurs) and derive aggregate supply of capital and aggregate demand for assets. The following proposition characterizes policy functions:

Proposition 2. *Decision rules and the value function of an i-producer that has productivity level* $A_I < A_I^*(q_S)$ *are:* $c = \frac{\phi}{1+\phi}\omega_{I_0}$, $k' = \frac{1}{1+\phi}\frac{(1-\delta)\omega_{I_0}}{q_S}$, $V^{I_0} = \Psi^{I_0}(K,E) + \left(1+\frac{1}{\phi}\right)\log\omega_{I_0}$ *where* $\omega_{I_0} = q_S k$ *and* $\phi = \frac{1-\beta}{\beta(\pi_I+\pi_C\cdot\mathbb{E}Z)}$. *Decision rules and the value function of an i-producer that has productivity level* $A_I \geq A_I^*(q_S)$ *are:* $c = \frac{\phi}{1+\phi}\omega_{I_P}$, $k' = \frac{1}{1+\phi}A_I\omega_{I_P}$, $V^{I_P} = \Psi^{I_P}(K,E,A_I) + \left(1+\frac{1}{\phi}\right)\log\omega_{I_P}$ *where* $\omega_{I_P} = q_S k$. *Decision rules and the value function of a c-producer are:* $c = \frac{\phi Z}{1+\phi Z}\omega_C$, $k' = \frac{1}{1+\phi Z}\frac{(1-\delta)\omega_C}{q_B}$, $V^C = \Psi^C(K,E,Z) + \left(Z+\frac{1}{\phi}\right)\log\omega_C$ *where* $\omega_C = (G(K)+q_B)\,k$.

Proposition 2 enables the derivation of the aggregate demand for labor, aggregate supply and aggregate demand for capital.

Aggregate demand for labor. Aggregation of (4) across the c-producers yields:

$$L_D(w(K),K) = \left(\frac{(1-\alpha)A_C}{w(K)}\right)^{1/\alpha}\cdot\pi_C\cdot K.$$

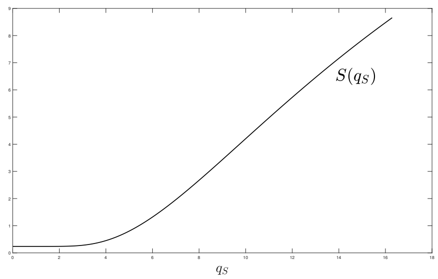
Function L_D is decreasing in wages and increases with technology level A_C.

Aggregate supply of capital. First observe that i-entrepreneurs with higher productivity (i.e., those $A_I \geq A_I^*(q_S)$) sell their entire capital. The total measure of those agents is $\pi_I\cdot\mathbb{P}_{A_I}\left(A_I \geq A_I^*(q_S')\right)$. There is $\pi_I\cdot\mathbb{P}_{A_I}\left(A_I < A_I^*(q_S')\right)$ of i-producers that set $i=0$. Individual supply of capital of the latter is:

$$k_S = k - \frac{k'}{1-\delta} = k - \frac{1}{1-\delta}\cdot\frac{1}{1+\phi}\cdot\frac{(1-\delta)}{q_S}\cdot q_S k = \frac{\phi}{1+\phi}k,$$

where Proposition 2 was used. Since the idiosyncratic shock that divides the pool of entrepreneurs into i-producers and c-producers is independent of individual capital holdings (shock A_I satisfies this property, too) then the following formula for the aggregate supply of capital $S(q_s)$ can be obtained:

Figure 2: Aggregate supply of capital (fixed K)



Notes: To plot the figure the following parameter values are used $\pi_I = 0.2$, $A_C = 1$, $Z_L = 1$, $Z_H = 3$, $\beta = 0.99$, $L = 100$, $\delta = 0.025$, $\alpha = 0.33$, $\pi(Z_H) = 0.3$, $\epsilon = 1.25$, investment opportunities are drawn from Gamma distribution featuring parameters 1 and 0.05. For Figures 4, 5 and of the fixed levels of K or E is the long run level of K (or E) after a long time of "normal times" ($Z = Z_L$).

$$S(q_s, K) = \left\{ \mathbb{P}_{A_I} \left(A_I < A_I^*(q_S) \right) \frac{\phi}{1+\phi} + \mathbb{P}_{A_I} \left(A_I \geq A_I^*(q_S) \right) \right\} \cdot \pi_I \cdot K. \qquad (5)$$

Since $0 < \phi/(1+\phi) < 1$ then $S_{q_S}(q_s, K) > 0$. This is because as q_S decreases then $A_I^*(q_S) = (1-\delta)/q_S$ falls and there is more i-entrepreneurs that sell their entire capital holdings. Observe that because density $f(A_I)$ was assumed to be continuous then by the Fundamental Theorem of Calculus $S_{q_S}(q_s, K)$ exists and is continuous in q_S. Notice that $\lim_{q_S \to 0} S(q_s, K) = [\phi/(1+\phi)] \cdot \pi_I \cdot K$, $\lim_{q_S \to +\infty} S(q_s, K) = \pi_I \cdot K$ for any value of K. The case of $\lim_{q_S \to 0} S(q_s, K) > 0$ seems to be surprising. This happens because if $q_S \to 0$ then $A_I^*(q_S) \to +\infty$ and the measure of i-producers that set $i = 0$ converges to π_I. Since they are not able to produce consumption goods they sell a non-zero proportion $\phi/(1+\phi)$ of their capital holdings. It is because they need to consume due to the logarithmic utility function. Figure 2 illustrates the aggregate supply of capital.

Aggregate demand for capital. Individual demand for capital of a c-entrepreneur can be derived using the formula for k' in Proposition 2:

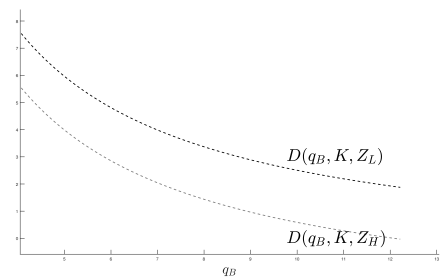
$$k_B = \frac{k'}{1-\delta} - k = k \cdot \left[\frac{1}{1+\phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1+\phi Z} - 1 \right].$$

Again, the assumption about the independence of idiosyncratic shocks of individual capital holdings is used which means that the aggregate demand for capital has the following form:

$$D(q_B, K, Z) = \left[\frac{1}{1+\phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1+\phi Z} - 1\right] \cdot \pi_C \cdot K. \quad (6)$$

It is clear that: $D_{q_B} < 0$ and $D_Z < 0$. The latter implies that an increase in Z translates into lower amount of assets demanded by c-producers. This, in turn, decreases the price of assets held by intermediaries (since the amount of capital is fixed as intermediaries buy assets from i-entrepreneurs during the first stage as it is shown in Figure 1).

Figure 3: Aggregate demand for capital (for low and high realizations of Z, fixed K)



Notes: To plot the figure the following parameter values are used $\pi_I = 0.2$, $A_C = 1$, $Z_L = 1$, $Z_H = 3$, $\beta = 0.99$, $L = 100$, $\delta = 0.025$, $\alpha = 0.33$, $\pi(Z_H) = 0.3$, $\epsilon = 1.25$, investment opportunities are drawn from Gamma distribution featuring parameters 1 and 0.05. For Figures 4, 5 and of the fixed levels of K or E is the long run level of K (or E) after a long time of "normal times" ($Z = Z_L$).

Observe that $\lim_{q_B \to 0} D(q_B, K, Z) = +\infty$ and $\lim_{q_B \to +\infty} D(q_B, K, Z) = -\frac{\phi Z}{1+\phi Z} \cdot \pi_C \cdot K < 0$. Additionally, if $q_B = q_B^0(K, Z) = \frac{G(K)}{\phi Z}$ then $D(q_B, K, Z) = 0$. The aggregate demand function is presented in Figure 3.

The aggregate law of motion for capital. To derive the aggregate law of motion for capital let us start with the investment decision made by i-entrepreneurs whose productivity satisfies $A_I \geq A_I^*(q_S)$. The amount of new capital generated by such entrepreneurs is (according to Proposition 2):

$$A_I \cdot i = k' = \frac{1}{1+\phi} \cdot A_I \cdot q_S \cdot k.$$

Aggregation across different values of A_I and individual capital holdings yields:

$$I(q_S, K) = \left[\int_{A_I^*(q_S)}^{+\infty} \frac{1}{1+\phi} \cdot A_I \cdot q_S \cdot f(A_I) dA_I\right] \pi_I \cdot K. \quad (7)$$

Let us analyze the effects of q_S on aggregate effective investment $I(q_S, K)$. Formula 7 shows that there are two forces at play. First, an increase in q_S causes a drop in $A_I^*(q_S)$. This means that more i-producers decide to sell their entire capital and invest a part $1/(1+\phi)$ of their proceedings (this is the extensive margin). Second, higher q_S boosts the i-entrepreneurs' wealth $\omega_{I_P} = q_S k$, so that they are able to get more consumption goods in exchange for the sold capital. Since their technology uses consumption goods to generate capital and since the proportion of goods that are invested $1/(1+\phi)$ is constant, then it leads to an increase in $I(q_S, K)$ (intensive margin). This means that q_S depends crucially on the condition of banks' balance sheets and hence the discussed effects constitute a mechanism through which financial disturbances are transmitted to the real economy.

Since the remaining capital depreciates at rate $0 < \delta < 1$ then the aggregate law of motion for K is:

$$K' = (1-\delta)K + I(q_S, K). \tag{8}$$

3.3 Intermediaries

Banks are competitive and take prices q_S and q_B as given. They begin the period with e consumption goods (also referred to as equity) - its level is determined by their decision in the previous period. During the first stage, they choose the level of capital k_F that they transfer from i-producers to c-producers. During the second stage (after the realization of Z), they choose consumption c and equity in the next period: e'. Since decisions are made in both stages, two maximization problems have to be specified. In the first stage, the bank solves:

$$W_1(e, K, E) = \max_{k_F} \mathbb{E}_Z \left(W_2(k_F, e, K, E, Z) \right), \tag{9}$$

where W_1 is the value function corresponding to the maximization problem solved in the first stage and W_2 is associated with the second stage problem that reads:

$$W_2(k_F, e, K, E, Z) = \max_{c, e'} \left\{ \log(c) + \beta W_1(e', K', E') \right\}$$

subject to:

$$\begin{cases} c + e' = e + \left(q_B(K, E, Z) - q_S(K, E) \right) \cdot k_F, \\ E' = E'(K, E, Z), \\ K' = K'(K, E). \end{cases} \tag{10}$$

The first constraint is the bank's budget constraint and it captures the implicit assumption that banks cannot default on their liabilities $q_S(K, E)k_F$.

It will become clear later (see the proof of Theorem 5) that in equilibrium both positive and negative spreads $q_B(K, E, Z) - q_S(K, E)$ are possible (where the sign depends on the realization of Z). This, in turn, implies that the decision about k_F is risky: not only does $q_B(K, E, Z)$ vary with Z but also $q_S(K, E) \cdot k_F$ that has

to be repaid to capital sellers at the end of the second stage is unaffected by the realization of aggregate uncertainty. This gives rise to the portfolio risk that is faced by intermediaries.
To obtain the analytical formulas for policies and value functions of the intermediary, the following condition needs to be satisfied:

A1 Aggregate shock Z takes two values: $Z \in \{Z_L, Z_H\}$. Probabilities $\mathbb{P}(Z = Z_L) = \pi(Z_L)$ and $\mathbb{P}(Z = Z_H) = \pi(Z_H)$ satisfy $\pi(Z_L) + \pi(Z_H) = 1$.

This assumption is made for the clarity of exposition (it is easier to analyze spreads in this case) and it simplifies the problem of the uniqueness of the Recursive Competitive Equilibrium (RCE) with the competitive banking sector. Observe, that since $D_Z < 0$ the state in which $Z = Z_H$ can be referred to as "crisis" in which demand for capital held by intermediaries drops (which causes a decrease in price $q_B(K, E, Z)$) and hence the value of banks' assets $q_B(K, E, Z) \cdot k_F$ falls.
Additionally, assumption A1 allows to transform (10) into a standard consumption-savings problem (that admits an analytical solution). It is because A1 implies that the optimal choice of k_F is a linear function of e, i.e.: $k_F = \Phi(K, E) \cdot e$. This means that the budget constraint in problem (10) takes the form that is analogous to the one in the standard consumption-savings problem:

$$c + e' = (1 + (q_B - q_S) \cdot \Phi(K, E)) \cdot e.$$

This is the main intuition behind the proof of the following proposition:

Proposition 3. *If A1 holds then the decision rules and value function of an intermediary are:* $c = (1 - \beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K, E, Z) + \frac{1}{1-\beta}\log\omega_F$, $k_F = \Phi(K, E) \cdot e$, *where* $\omega_F = e + (q_B - q_S)k_F$.

Proposition 3 will be useful when characterizing the equilibrium allocation with competitive banking sector. The exact analytic formula for $\Phi(K, E)$ is presented in Appendix B. It is easy to check that $k_F = \Phi(K, E) \cdot e$ can be rewritten as:

$$\pi(Z_L) \cdot \frac{q_B(K, E, Z_L) - q_S(K, E)}{e + [q_B(K, E, Z_L) - q_S(K, E)]\, k_F} + \\ + \pi(Z_H) \cdot \frac{q_B(K, E, Z_H) - q_S(K, E)}{e + [q_B(K, E, Z_H) - q_S(K, E)]\, k_F} = 0, \quad (11)$$

which is the FOC characterizing the optimal choice of k_F during the first stage.

3.4 Equilibrium

In this subsection the definition of recursive competitive equilibrium is presented. To distinguish between consumption and capital choices of different types of agents, subscripts I_P, I_0, C and F (for i-producer with $A_I \geq A_I^*(q_S)$, i-producer with $A_I < A_I^*(q_S)$, c-producer and financial intermediary, respectively) are used.

Definition 4. Recursive Competitive Equilibrium with a competitive banking sector consists of: pricing functions $q_B(K,E,Z)$, $q_S(K,E)$, $w(K)$, the perceived law of motion for intermediaries' equity $E'(K,E,Z)$ and aggregate capital $K'(K,E)$, decision rules $k_F(e,K,E)$, $e'(e,K,E,Z)$, $c_F(e,K,E,Z)$, $c_C(k,K,E,Z)$, $k'_C(k,K,E,Z)$, $k_B(k,K,E,Z)$, $l(k,K,E,Z)$, $c_{I_P}(k,K,E,A_I)$, $k'_{I_P}(k,K,E,A_I)$, $i(k,K,E,A_I)$, $c_{I_0}(k,K,E)$, $k'_{I_0}(k,K,E)$, $k_S(k,K,E)$, value functions $W_1(e,K,E)$, $W_2(k_F,e,K,E,Z)$, $V^C(k,K,E,Z)$, $V^{I_P}(k,K,E,A_I)$, $V^{I_0}(k,K,E)$ and the stochastic processes determining the evolution of Z, A_I, and the producer's type over time, such that:
1) Decision rules $k_F(e,K,E)$, $e'(e,K,E,Z)$, $c_F(e,K,E,Z)$ and $W_1(e,K,E)$, $W_2(k_F,e,K,E,Z)$ solve the dynamic problem of the financial intermediary given $q_B(K,E,Z)$, $q_S(K,E)$, $E'(K,E,Z)$, $K'(K,E)$, and the stochastic processes,
2) Decision rules $c_C(k,K,E,Z)$, $k'_C(k,K,E,Z)$, $k_B(k,K,E,Z)$, $l(k,K,E,Z)$ and value functions $V^C(k,K,E,Z)$, $V^{I_P}(k,K,E,A_I)$, $V^{I_0}(k,K,E)$ solve the dynamic problem of the c-producer given: $q_B(K,E,Z)$, $w(K)$, $E'(K,E,Z)$, $K'(K,E)$, and the stochastic processes,
3) Decision rules $c_{I_P}(k,K,E,A_I)$, $k'_{I_P}(k,K,E,A_I)$, $i(k,K,E,A_I)$ and value functions $V^C(k,K,E,Z)$, $V^{I_P}(k,K,E,A_I)$, $V^{I_0}(k,K,E)$ solve the dynamic problem of the i-producer for which $A_I \geq A_I^*(q_S(K,E))$ given $q_S(K,E)$, $E'(K,E,Z)$, $K'(K,E)$, and the stochastic processes,
4) Decision rules $c_{I_0}(k,K,E)$, $k'_{I_0}(k,K,E)$, $k_S(k,K,E)$ and value functions $V^C(k,K,E,Z)$, $V^{I_P}(k,K,E,A_I)$, $V^{I_0}(k,K,E)$ solve the dynamic problem of the i-producer for which $A_I < A_I^*(q_S(K,E))$ given $q_S(K,E)$, $E'(K,E,Z)$, $K'(K,E)$ and the stochastic processes,
5) Consistency conditions hold: $e'(e,K,E,Z) = E'(K,E,Z)$ and

$$K'(K,E) = (1-\delta)K + I(q_S(K,E),K),$$

where:

$$I(q_S(K,E),K) = \left[\int_{A_I^*(q_S(K,E))}^{+\infty} i(k,K,E,A_I)\cdot f(A_I)dA_I\right]\pi_I.$$

6) Markets clear, i.e.: $k_F(e,K,E) = S(q_S(K,E),K)$, $L_D(w(K),K) = L$, $k_F(e,K,E) = D(q_B(K,E,Z),K,Z)$.

Characterization. Suppose that values are K and E are given. This paragraph discusses how the values K' and E' can be obtained using the model's equations and the current realization of Z.
First, notice that we can use the labor market clearing condition $L_D(w(K),K) = L$ to get the expression for wages:

$$w(K) = (1-\alpha)\cdot A_C\cdot(\pi_C\cdot K)^\alpha\cdot L^{-\alpha}. \tag{12}$$

Let us combine the bank's FOC (11) with the reformulated market clearing conditions, and the consistency condition $E = e$ (the arguments of the policy and pricing functions are suppressed to economize on the notation and the fact that S^{-1} with respect to its first argument exists is used as S is strictly increasing and similar arguments are applied to D^{-1}):

$$\pi(Z_L) \cdot \frac{D^{-1}\left(S(q_S, K), K, Z_L\right) - q_S}{E + \left[D^{-1}\left(S(q_S, K), K, Z_L\right) - q_S\right] S(q_S)} + \\ + \pi(Z_H) \cdot \frac{D^{-1}\left(S(q_S, K), K, Z_H\right) - q_S}{E + \left[D^{-1}\left(S(q_S, K), K, Z_H\right) - q_S\right] S(q_S)} = 0. \quad (13)$$

Or, shortly:

$$\mathbb{E}_Z \left(\frac{D^{-1}\left(S(q_S, K), K, Z\right) - q_S}{E + \left[D^{-1}\left(S(q_S, K), K, Z\right) - q_S\right] S(q_S)} \right) = 0. \quad (14)$$

Observe that the LHS of (14) is a function of q_S and state variables E and K which are taken as given in the current period. This means that (13) can be used to pin down the equilibrium value of q_S. Given q_S, we can calculate $I(q_S, K)$ and the next period value of aggregate capital: K'.

Once we have computed q_S, we are able to calculate k_F (from the market clearing condition $k_F = S(q_S, K)$). Given the current realization of Z, $q_B = D^{-1}\left(S(q_S, K), K, Z\right)$ we are able to pin down the banker's wealth ω_F which, together with Proposition 3, enables to compute E'. This means that it is possible to calculate the path of endogenous state variables analytically, without the need of using any global or local solution methods. The price-formation in equilibrium is presented in Figure 4.

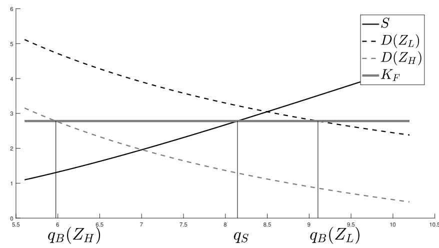
It is clear that if the solution to (13) exists and is unique then the values of K' and E' are well-defined (i.e., there is only one pair that is consistent with the equilibrium path) given K, E, Z. This, in turn, implies the existence and uniqueness of the RCE described in Definition 4. This result follows because given the existence and uniqueness of q_S that solves (14) (given values $K \in \mathcal{K}$ and $E \in \mathcal{E}$, where $\mathcal{K}$ and $\mathcal{E}$ are spaces of state variables) we are able to compute $q_B(K, E, Z_L)$ and $q_B(K, E, Z_H)$. In other words, for all $K \in \mathcal{K}$ and $E \in \mathcal{E}$ the dynamic programming problem described by (9) and (10) is well-defined as we know the prices that are taken as given by the intermediary. It is therefore sufficient to apply the standard fixed-point argument (the Banach theorem) to the dynamic programming problem characterized by (9) and (10) to argue that its solution exists and is unique.

Let us turn to the main result of this section:

Theorem 5. *If A1 holds then the solution to Equation* (13) *exists and is unique. Moreover, the aggregate reallocation of capital* k_F *increases with* E.

Observe that this result gives rise to a direct link between the condition of banks' balance sheets and the amount of capital reallocated in the economy, which is discussed in a more detailed way in the next paragraph.

Figure 4: Price formation in equilibrium



Notes: Solid lines denote decisions/objects resulting from the choices made in the first stage of the period and dashed lines denote the objects that are determined in the second stage. Supply curve S corresponds to the one depicted in Figure 2. Demand curves associated with the two realizations of the shock correspond to those presented in Figure 3. To plot the figure the following parameter values are used $\pi_I = 0.2$, $A_C = 1$, $Z_L = 1$, $Z_H = 3$, $\beta = 0.99$, $L = 100$, $\delta = 0.025$, $\alpha = 0.33$, $\pi(Z_H) = 0.3$, $\epsilon = 1.25$, investment opportunities are drawn from Gamma distribution featuring parameters 1 and 0.05. For Figures 4, 5 and of the fixed levels of K or E is the long run level of K (or E) after a long time of "normal times" ($Z = Z_L$).

Transmission mechanism. Let us discuss the channels through which changes in Z affect the economy. Let us consider the situation at the end of the first stage, i.e., before the realization of Z. Observe that k_F is already chosen by banks and hence the value of deposits that needs to be repaid in the second stage - $q_S \cdot S(q_S, K)$ is fixed, too. Since q_S is defined in the first stage as well, then K' will remain unaffected by Z (see Equation (8)).
Suppose that the current realization of Z is $Z = Z_H$. By the proof of Theorem 5, this implies that $q_B(K, E, Z_H) < q_S$. Since k_F is already fixed, the value of ω_F drops (i.e., the financial wealth of intermediaries falls). Since $e' = \beta\omega_F$ (by Proposition 3), then lower ω_F translates into a decreased level of banks' equity in the next period. This, in turn, has adverse effects on the amount of intermediated capital k'_F in the next period (by Theorem 5): k'_F decreases and the market clearing condition for "deposits" $k_F = S(q'_S)$ implies that q'_S falls. This has two effects: first, i-producers obtain less consumption goods $q'_S \cdot S(q'_S)$ that can be transformed into capital. Second, since q'_S is lower $A^*_I(q'_S)$ grows and the proportion of i-producers that produce investment goods $\mathbb{P}(A_I \geq A^*_I(q'_S))$ falls. Both factors mean that I' is lower and hence the level of capital in the subsequent period K'' deteriorates which means that the aggregate output of consumption goods - i.e., $A_C\left(\pi_C K''\right)^{\alpha} L^{1-\alpha}$ is lower. It is therefore clear that the condition of banks' balance sheets - e' is the only channel through which a decrease in demand for capital held by intermediaries in the current period (caused by $Z = Z_H$) negatively and persistently affects the real economy.

The role of intermediaries. This section finishes with a comment on the role played by the intermediaries. Notice that the decision about k_F is risky as it is made before the realization of Z. This means that intermediaries absorb the risk that would be otherwise faced by the capital sellers (i-entrepreneurs). More precisely, if banks were absent then capital sellers would sell capital to c-producers after the realization of Z and the price of this transaction would depend on the realization of aggregate uncertainty. If, however, banks are in place then i-entrepreneurs are insured against shifts in asset prices as they are offered riskless "deposit" contracts so that they purchase $q_S S(q_S)$ of consumption goods at price q_S that is independent of the current realization of aggregate uncertainty Z.

4 Monopolistically competitive intermediaries

This section studies the economy in which intermediaries have a certain degree of monopoly power. A standard construction called the Dixit-Stiglitz aggregator is used and applied to the market on which capital is sold to c-producers. Such a formalization of the intermediaries' monopoly power enables to calculate explicit formulas for banks' policy functions and compare it to the model with perfectly competitive banks. It is because the amount of channeled resources k_F remains a linear function of equity e and thus the bank's budget constraint is linear in equity so that we can use the results shown by Alvarez and Stokey (1998) again.
Observe that the considerations about the intermediaries' market structure do not affect the producers' sector - this implies that entrepreneurs' policy functions, aggregate demand for capital $D(q_S, K, Z)$ and aggregate supply $S(q_S, K)$ of assets remain unchanged.

4.1 Capital retailers

To construct the monopolistically competitive market structure, a new type of agent to the model is introduced: perfectly competitive retailers that earn zero profits, buy capital from monopolistic intermediaries and sell it to c-entrepreneurs. This modification allows for replacing perfect competition with a monopolistic competition (in the market where capital is sold by intermediaries) in a standard and tractable way. Moreover, as discussed below, introducing those auxiliary agents enables me to nest the model with perfect competition in the one where intermediaries feature certain monopoly power (i.e., are monopolistically competitive).
It is assumed that all actions discussed in this subsection take place in the second stage. To produce capital good k_F, a retailer must purchase a continuum of differentiated wholesale capital goods $k_{F,j}$ indexed by $j \in [0, 1]$ (where j is an index assigned to a single banker). Retail good can be treated as a bundle/package of wholesale assets. Additionally, capital provided by intermediaries is differentiated and hence various capital goods are imperfect substitutes. This idea is formalized by

the Dixit-Stiglitz aggregator:

$$k_F = \left[\int_0^1 k_{F,j}^{1/\epsilon} dj\right]^\epsilon, \ \epsilon > 1, \tag{15}$$

where $\epsilon > 1$ measures the substitutability between different "varieties of capital" supplied by intermediaries. The profit function of the retailer reads:

$$q_B k_F - \int_0^1 q_{B,j} k_{F,j} dj. \tag{16}$$

Plugging (15) into (16) and deriving the FOC with respect to $k_{F,j}$ good yields:

$$q_B \left[\int_0^1 k_{F,j}^{1/\epsilon} dj\right]^{\epsilon-1} k_{F,j}^{(1/\epsilon)-1} = q_{B,j}.$$

(15) is used again to get the demand for capital of banker j:

$$k_{F,j} = \left(\frac{q_{B,j}}{q_B}\right)^{\epsilon/(1-\epsilon)} k_F. \tag{17}$$

Relationship described by (17) is taken as given by the monopolistic intermediary.

4.2 Monopolistic intermediaries

Bankers purchase capital in a perfectly competitive market and sell it to retailers in a monopolistically competitive environment. In the first stage, bank j solves:

$$W_1(e, K, E) = \max_{k_{F,j}} \mathbb{E}_Z \left(W_2(k_{F,j}, e, K, E, Z)\right), \tag{18}$$

and the second stage problem reads:

$$W_2(k_{F,j}, e, K, E, Z) = \max_{c,e'} \{\log(c) + \beta W_1(e', K', E')\}$$

subject to:

$$\begin{cases} c + e' = e + \left[q_B(K, E, Z) \cdot \left(\frac{k_F}{k_{F,j}}\right)^{1-1/\epsilon} - q_S(K, E)\right] k_{F,j}, \\ E' = E'(K, E, Z), \\ K' = K'(K, E), \end{cases} \tag{19}$$

where a reformulated version of (17), i.e., $q_{B,j} = q_B \cdot \left(\frac{k_F}{k_{F,j}}\right)^{1-1/\epsilon}$, has been plugged into the budget constraint. Following the literature on the monopolisitically competitive market structures, let me concentrate on the symmetric case in which $k_F = k_{F,j}$ (the so-called symmetric equilibrium). The following proposition characterizes policy functions of the monopolistic intermediary:

Proposition 6. *If A1 holds then decision rules and value function of the monopolistic intermediary are:* $c = (1-\beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K,E,Z) + \frac{1}{1-\beta}\log\omega_F$, $k_{F,j} = \tilde{\Phi}(K,E)\cdot e$, *where* $\omega_F = e + (q_B - q_S)\,k_{F,j}$.

The analytical expression for $\tilde{\Phi}$ is presented in Appendix B.

4.3 Equilibrium

The full definition of Recursive Competitive Equilibrium with monopolistically competitive banks is not presented – it is analogous to the case with perfectly competitive intermediaries. Moreover, calculating K' and E' given K, E and Z requires analogous steps as in the case of the economy with competitive banks. Similarly to the case discussed in Section 3, equation that combines the bank's FOC (with respect to k_F) and the market clearing conditions plays a crucial role. The following formula is an equivalent of Equation (13) adopted to the environment with monopolistically competitive banks:

$$\pi(Z_L)\cdot\frac{\frac{1}{\epsilon}D^{-1}\left(S(q_S(K,E)),K,Z_L\right)-q_S(K,E)}{e+\left[D^{-1}\left(S(q_S(K,E)),K,Z_L\right)-q_S(K,E)\right]S(q_S(K,E))}+$$
$$+\,\pi(Z_H)\cdot\frac{\frac{1}{\epsilon}D^{-1}\left(S(q_S(K,E)),K,Z_H\right)-q_S(K,E)}{e+\left[D^{-1}\left(S(q_S(K,E)),K,Z_H\right)-q_S(K,E)\right]S(q_S(K,E))}=0. \quad (20)$$

Observe, that the only difference between (13) and (20) is presence of fraction $\frac{1}{\epsilon}$ in (20). In other words, the allocation associated with monopolictic banks becomes identical to the one associated with the model with perfectly competitive banks as capital goods become perfect substitutes, i.e. when $\epsilon \to 1$.
This section finishes with the analog of Theorem 5 in the model with monopolistically competitive intermediaries:

Theorem 7. *Under A1, the solution to Equation* (20) *exists and is unique. Additionally, in the RCE with monopolistically competitive intermediaries aggregate reallocation of capital* k_F *increases with* E.

5 Comparison of the economies with competitive and monopolistically competitive intermediaries

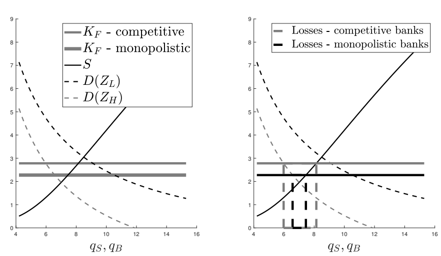
This part compares two economies – the one with competitive banks and the one monopolistically competitive intermediaries. I particular, it articulates the potential advantages and disadvantages of the competitive banking sector in comparison to the monopolistic industry in the short run defined as a situation when state variables K and E are the same in both economies and only the current period is analyzed. The long-run trade-off (associated with the features of ergodic/stationary distributions of K and E in both economies) is discussed in Appendix A.

Competition and the amount of intermediated capital. Suppose that both economies have the same initial value of aggregate banks' equity - E and the same aggregate capital stock K. The next proposition characterizes the relationship between k_F^C and k_F^{MC}.

Proposition 8. *If the initial value of aggregate intermediaries' equity E and aggregate capital K are the same in both economies: the one with competitive banks and the one with monopolistically competitive intermediaries, then the amount of intermediated capital is strictly higher in the economy with competitive banks.*

This result is illustrated in the left panel of Figure 5.

Figure 5: Capital intermediated in economy with monopolistically and in economy with competitive banks



Notes: Solid lines denote decisions/objects resulting from the choices made in the first stage of the period and dashed lines denote the objects that are determined in the second stage. Rectangles in the right panel are losses generated by banks in the case of the arrival of an adverse shock. To plot the figure the following parameter values are used $\pi_I = 0.2$, $A_C = 1$, $Z_L = 1$, $Z_H = 3$, $\beta = 0.99$, $L = 100$, $\delta = 0.025$, $\alpha = 0.33$, $\pi(Z_H) = 0.3$, $\epsilon = 1.25$, investment opportunities are drawn from Gamma distribution featuring parameters 1 and 0.05. For Figures 4, 5 and of the fixed levels of K or E is the long run level of K (or E) after a long time of "normal times" ($Z = Z_L$).

Banks' losses in the crisis. Again, consider the situation when both economies have the same initial stock of banks' equity – E and capital K. Recall that in the economy with competitive banks (by the proof of Theorem 5):

$$q_B^C(Z_H) - q_S^C = D^{-1}(S(q_S^C), K, Z_H) - q_S^C < 0,$$

where the dependence of pricing functions q_B and q_S on K and E is suppressed for the notational convenience. This means that losses incurred by competitive banks when $Z = Z_H$ are:

$$L^{CE}(Z_H) = \left(q_B^C(Z_H) - q_S^C\right) \cdot S(q_S^C) < 0. \tag{21}$$

Let us compare (21) with the losses generated by monopolistic intermediaries. There are two effects that magnify the losses of competitive industry in comparison to the monopolistically competitive bankers. First, by Proposition 8 and by the market clearing condition for "deposits" we get $S\left(q_S^C\right) > S\left(q_S^{MC}\right)$ and hence:

$$q_B^C(Z_H) = D^{-1}\left(S\left(q_S^C\right), K, Z_H\right) < D^{-1}\left(S\left(q_S^{MC}\right), K, Z_H\right) = q_B^{MC}(Z_H).$$

This together with the fact that $q_S^C > q_S^{MC}$ implies:

$$q_B^C(Z_H) - q_S^C < q_B^{MC}(Z_H) - q_S^{MC} < 0. \tag{22}$$

Inequality (22) means that one reason for which competitive intermediaries generate higher losses than monopolistic banks is due to the fact that they do not internalize the influence of their portfolio decisions (i.e., the decision about k_F^C) on prices. Second, since $S\left(q_S^C\right) > S\left(q_S^{MC}\right)$ the uninternalized effect on prices is amplified even further which means that:

$$L^C(Z_H) = \left(q_B^C(Z_H) - q_S^C\right) S\left(q_S^C\right) < \left(q_B^{MC}(Z_H) - q_S^{MC}\right) S\left(q_S^{MC}\right) = L^{MC}(Z_H).$$

These considerations are summarized by the following proposition:

Proposition 9. *If the initial value of aggregate intermediaries' equity E and the capital stock K are the same in both economies: the one with competitive banks and the one with monopolistically competitive intermediaries, then the aggregate losses generated by banks for $Z = Z_H$ (i.e. "crisis") are higher in the economy with competitive intermediaries.*

Proposition 9 has an important dynamic consequence: if $Z = Z_H$ occurs in the initial period then $\omega_F^{MC} > \omega_F^C$ and hence monopolistically competitive banks accumulate higher equity E' than competitive banks. This, coupled with the results presented in Theorems 5 and 7, means that the amount of capital transferred from i-producers to c-entrepreneurs in the subsequent period can be strictly lower for the economy with competitive banks than in economy with monopolistically competitive intermediaries. These considerations are illustrated in the right panel of Figure 5.

6 Conclusions

This paper presents a tractable dynamic general equilibrium model with financial sector and applied it to study the business cycle consequences of changes in the competition in the financial sector. The model is used to investigate the dynamic properties of two regimes: the one with competitive banks and the second with monopolistically competitive intermediaries.

More precisely, the paper has concentrated on two time horizons: the short-run perspective and the long-run perspective. The first one indicated that competitive banking industry guarantees higher level of intermediation activities but, at the same time, it exhibits higher exposure to aggregate risk (losses generated by competitive banks are larger than those incurred by monopolistic intermediaries). Therefore if an adverse aggregate shock arrives, equity of competitive banks is drained more severely, which impedes intermediation in subsequent periods. This, in turn, means that negative impact of monopolistic wedge on the amount of channeled funds can be outweighed by an increased intermediation ability of monopolistic banks during economic downturns.

The long-run perspective concerned the analysis of ergodic distributions of aggregate variables. In particular, it has been shown that the short-run trade-off has its counterpart in the long-run: on the one hand ergodic density of capital (and output) under competitive regime has its upper bound shifted to the right in comparison to the upper bound of the density associated with monopolistic regime. The opposite relationship is true for the upper bounds of ergodic densities of banks' equity. This has an important consequence: higher equity cushion of monopolistic banks absorbs adverse aggregate shocks more effectively which implies lower aggregate uncertainty induced by the monopolistic financial sector.

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References

[1] Alvarez F., Stokey N., (1998), Dynamic Programming with Homogeneous Functions, *Journal of Economic Theory* 82(1), 167–189.

[2] Berentsen A., Camera G., Waller C., (2007), Money, credit and banking, *Journal of Economic Theory* 135(1) 171–195.

[3] Bernanke B., Gertler M., (1989), Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review* 79(1), 14–31.

[4] Bigio S., d'Avernas A., (2021), Financial Risk Capacity, *American Economic Journal: Macroeconomics* 13(4), 142–181.

[5] Boyd J.H., De Nicolo G., (2005), The Theory of Bank Risk Taking and Competition Revisited, *Journal of Finance* 60(3), 1329–1343.

[6] Brunnermeier M., Sannikov Y., (2014), A Macroeconomic Model with a Financial Sector, *American Economic Review* 104(2), 379–421.

[7] Gu C., Mattesini F., Wright R., (2016), Money and Credit Redux, *Econometrica* 84, 1–32.

[8] Jermann U., Quadrini V., (2012), Macroeconomic Effects of Financial Shocks, *American Economic Review* 102(1), 238–271.

[9] Keeley M., (1990), Deposit Insurance, Risk, and Market Power in Banking, *American Economic Review* 80(5), 1183–1200.

[10] Kiyotaki N., Moore J., (2019), Liquidity, Business Cycles and Monetary Policy, *Journal of Political Economy* 127(6).

[11] Lagos R., Wright R., (2005), A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy* 113(3), 463–484.

[12] Martinez-Miera D., Repullo R., (2010), Does Competition Reduce the Risk of Bank Failure?, *Review of Financial Studies* 23(10), 3638–3664.

[13] Stiglitz J., Weiss A., (1981), Credit Rationing in Markets with Imperfect Information, *American Economic Review* 71(3), 393–410.

Appendix A The long-run trade-off, discussion of the assumptions, inefficiency of the RCE allocation

Appendix A.1 The long-run trade-off

This subsection analyzes the ergodic distributions of K and E:

$$\mu_K(\mathcal{K}) = \lim_{T\to+\infty}\frac{1}{T}\cdot\sum_{t=0}^{T}\mathbb{I}_{\{K_t\in\mathcal{K}\}},$$
$$\mu_E(\mathcal{E}) = \lim_{T\to+\infty}\frac{1}{T}\cdot\sum_{t=0}^{T}\mathbb{I}_{\{E_t\in\mathcal{E}\}},$$

where $\mathcal{E}$ and $\mathcal{K}$ are measurable sets under two different regimes: perfectly competitive and monopolistically competitive banks. In other words, the properties of the average distribution of sequences $\{K_t\}_{t=0}^{+\infty}$, $\{E_t\}_{t=0}^{+\infty}$ are studied (denoted by measures μ_K and μ_E,respectively) generated by an infinite sequence of exogenous stochastic realizations $\{Z_t\}_{t=0}^{+\infty}$.

First, the analytic characterization of the upper and lower bounds of the support of ergodic densities associated with distributions μ_K and μ_E is presented. Second, numerical simulations are used to explore some additional features of those distributions that are tightly associated with the results concerning the bounds. First of all, however, let us modify the model to make the analysis more tractable. In particular, to simplify the exposition, it is assumed that $\mathbb{P}(A_I = 1) = 1$, i.e. all i-producers have the same level of productivity. This assumption holds throughout this section.

To guarantee that equilibrium with $\mathbb{P}(A_I = 1) = 1$ exists, it is assumed that parameters satisfy the following inequality:

$$\left[\frac{(1+\phi Z_L)\frac{\pi_I}{\pi_C}+\phi Z_L}{(1+\phi Z_H)\frac{\pi_I}{\pi_C}+\phi Z_H}\right]\frac{1+\phi}{\pi_I} > \frac{1}{\delta}-1. \tag{A1}$$

It is easy to see that the set of parameters satisfying A1 is non-empty – it is because the LHS of condition (A1) is always strictly positive and the limit of the RHS when $\delta\to 1$ is zero. First, notice that the necessary condition for the existence of equilibrium is:

$$\forall_K \frac{G(K)}{(1+\phi Z_H)\frac{\pi_I K}{\pi_C K}+\phi Z_H} > 1-\delta. \tag{A2}$$

The LHS of (A2) is the inverse demand function evaluated at $\pi_I K$ (the amount of capital supplied by i-entrepreneurs when $q_S(K,E) > 1-\delta$). Condition (A2) says that the aggregate demand curve for the capital channeled by banks (that corresponds to realization Z_H of the aggregate shock) intersects the $S(q_S, K)$ scheme for such value of q_S that $S(q_s, K) = \pi_I \cdot K > 0$. It is because the situation in which $D(q_B, K, Z_H)$ and $S(q_s, K)$ cross each other at $q_B = q_S < 1 - \delta$ if $Z = Z_H$ has to be excluded (which would imply that the supply of capital is 0). The following lemma shows that (A2) is true when condition (A1) is satisfied:

Lemma 10. *Condition* (A2) *holds if parameters satisfy* (A1).

The economy with $\mathbb{P}(A_I = 1) = 1$ is described in Appendix A in a more detailed manner. It is easy to extend those results to describe the model with $\mathbb{P}(A_I = 1) = 1$ and $\epsilon > 1$ (i.e., with monopolistically competitive intermediaries).

Let us start with the lower bounds on the supports of ergodic densities of K^C, K^{MC}, E^C and E^{MC} (these variables denote aggregate capital in economy with competitive banks, aggregate capital in economy with monopolistically competitive banks, aggregate equity in economy with competitive banks, aggregate equity in economy with monopolistically competitive banks, respectively). Observe that the existence of ergodic densities is assumed. If they do not exist (see for example the Radon-Nikodym theorem) then all results in this section can be reformulated in terms of probability measures which is always possible.

To put it differently, lower bounds on densities' supports define threshold values below which K^C, K^{MC}, E^C and E^{MC} never fall if the economy starts from the point characterized with a positive ergodic density. It is easy to show that the following proposition holds:

Proposition 11. *The common lower bound on the supports of ergodic densities associated with E^C and E^{MC} is $\underline{E} = 0$.*

To obtain this result the Borel-Cantelli lemma is used and the law of motion for E. The next proposition, that characterizes the lower bounds for K^C, K^{MC}, requires some more refined arguments than those used in the proof of Proposition 11:

Proposition 12. *If $\mathbb{P}(A_I = 1) = 1$ and condition* (A1) *hold then the common lower bound on the supports of ergodic densities associated with K^C and K^{MC} is $\underline{K} = \left(\frac{\Psi}{\delta}\right)^{1/(1-\alpha)}$ where Ψ is a function of parameters.*

One remark is in order. Since the probability of the crisis $\pi(Z_H)$ is in general (i.e. for reasonable parametrization of the model) significantly lower than the probability of a "good" shock $\pi(Z_L)$ then the chance that the aggregate level of capital approaches to $\underline{K}$ is extremely low. This, in turn, means that the value of $\underline{K}$ has a negligible influence on the moments associated with ergodic distributions of K^C and K^{MC}. It is therefore much more important to study the upper bounds on K and E. The next proposition establishes the relationship between the upper bounds on $K^C$and K^{MC} (let us denote them by $\bar{K}^C$ and $\bar{K}^{MC}$):

Proposition 13. *If $\mathbb{P}(A_I = 1) = 1$ and condition* (A1) *hold then $\frac{d\bar{K}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ is negative.*

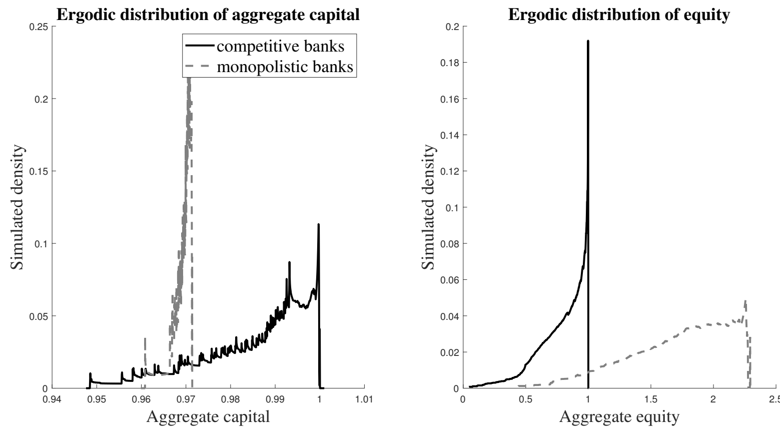
Proposition 13 says that the upper bound of the long-run distribution of capital decreases when perfectly competitive market becomes monopolistic. On the one hand it is intuitive because if $\epsilon > 1$ then intermediaries increase their profits and transfer less resources to the investors that create new capital. On the other hand, one could argue that this effect can be mitigated (or even eliminated) because if banks have higher profits then their long-run equity is higher, too (this intuition is confirmed by Proposition 14). This, in turn, together with Theorem 7, could imply that the negative effect of the growth in ϵ could be outweighed by the impact of higher equity. Proposition 13 states that this potentially mitigating effect is too weak and hence $\bar{K}^{MC}$ decreases in ϵ.
The next proposition describes the impact of ϵ on $\bar{E}^{MC}$. To prove this statement it is sufficient to assume one additional requirement, i.e. that $\pi(Z_L)\beta > \alpha$ holds:

Proposition 14. *If $\pi(Z_L)\beta > \alpha$, $\mathbb{P}(A_I = 1) = 1$ and condition* (A1) *hold then $\frac{d\bar{E}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ is positive.*

Again, Proposition 14 shows which of the two forces affecting $\bar{E}^{MC}$ is stronger when ϵ increases. The first force increases banks' profits when ϵ grows because intermediaries have a stronger impact on prices q_B^{MC}. The second effect implies that if ϵ increases then (by Proposition 13) average K^{MC} drops and hence the amount of intermediated capital k_F^{MC} is lower. This affects intermediaries' profits $\left(q_B^{MC}(Z) - q_S^{MC}\right) k_F^{MC}$ in a negative way. Proposition 14 shows that the latter effect is dominated by the first one. This in turn means that monopolistically competitive industry accumulates higher equity buffer against adverse aggregate shocks.

To illustrate the consequences of Propositions 11-14, let us use numerical simulations. Results are shown in Figure A1. For the clarity of exposure, the values of aggregate variables are standardized: aggregate capital is divided by the upper bound $\bar{K}^C$ and aggregate equity is divided by $\bar{E}^C$ (i.e., the standardized values of the upper bounds are denoted by: $\bar{K}^C_{std} = \frac{\bar{K}^C}{\bar{K}^C} = 1$, $\bar{K}^{MC}_{std} = \frac{\bar{K}^{MC}}{\bar{K}^C}$, $\bar{E}^C_{std} = \frac{\bar{E}^C}{\bar{E}^C} = 1$, $\bar{E}^{MC}_{std} = \frac{\bar{E}^{MC}}{\bar{E}^C}$). Simulation confirms the results presented in Propositions 13 and 14. The upper bound on K is higher in the economy with perfectly competitive banks and the upper bound on E is higher for the economy with monopolistically competitive banks. As expected, ergodic densities exhibit a significant concentration in the neighborhood of the upper bounds since $\pi(Z_H) < \pi(Z_L)$.
Observe that the fact that $\bar{E}^C < \bar{E}^{MC}$ (this relationship is certainly inherited by the means of ergodic distributions) has an additional, important consequence. Since the aggregate equity of banks tends to be higher in the economy with monopolistic intermediaries, the financial system has a greater capacity to absorb adverse shocks Z_H in that case. Hence, not only is the variance of K (and hence the variance of output because labor supply is assumed to be inelastic) significantly lower in the economy with monopolistic banks but also recessions experienced by the economy with perfectly competitive banks are more severe.

Figure A1: Ergodic distributions



Notes: To plot Figure A1 the following parameter values are used $\pi_I = 0.2$, $A_C = 1$, $Z_L = 1$, $Z_H = 10$, $\beta = 0.99$, $L = 100$, $\delta = 0.025$, $\alpha = 0.33$, $\pi(Z_H) = 0.01$, $\epsilon = 1.025$, investment opportunities are drawn from Gamma distribution featuring parameters 1 and 0.05.

Appendix A.2 Derivations of the results related to the long-run trade-off

Let us describe how the economy with $\mathbb{P}(A_I = 1) = 1$ looks like. Lemma 1 is used to conclude that all i-entrepreneurs invest only if:

$$q_S(K, E) \geq 1 - \delta.$$

If this condition does not hold the none of them invest. This implies that the capital supply function takes the following form:

$$S(q_s, K) = \begin{cases} \pi_I \cdot K, & \text{if } q_S(K, E) \geq 1 - \delta, \\ 0, & \text{otherwise.} \end{cases} \tag{A3}$$

Observe that (A3) and the market clearing for "deposits" imply that the amount of intermediated capital is not dependent on E. The problem of c-producer remains unchanged so aggregate demand for capital is:

$$D(q_B, K, Z) = \left[\frac{1}{1 + \phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1 + \phi Z} - 1 \right] \cdot \pi_C \cdot K.$$

Since $S(q_S, K)$ is not a continuous function then we need an additional argument to show that equilibrium exists if $\mathbb{P}(A_I = 1) = 1$. This condition in shown in the main text, and is summarized by Lemma 10.

Intermediaries solve the same problem as before. We are in position to prove existence and uniqueness of equilibrium in the simplified environment. Similarly to the more general case the equilibrium condition (i.e. bank's FOC combined with market clearing for deposits and capital sold to c-entrepreneurs) plays crucial role (recall that if (A1) holds then $S(q_S, K) = S(K) = \pi_I K$; this implies, additionally, that inverse demand function D^{-1} is independent of E):

$$\pi(Z_L) \cdot \frac{D^{-1}(K, Z_L) - q_S}{E + [D^{-1}(K, Z_L) - q_S]\,\pi_I K} + \\ + \pi(Z_H) \cdot \frac{D^{-1}(K, Z_H) - q_S}{E + [D^{-1}(K, Z_H) - q_S]\,\pi_I K} = 0. \quad \text{(A4)}$$

The following theorem shows that q_S that solves (A4) exists and is unique.

Theorem. *If* (A1) *holds then solution to Equation* (A4) *exists and is unique.*

It is clear that the amount of reallocated capital is independent of E as it is always equal to $\pi_I K$. This means that result analogous to Theorem 5 does not hold. It does not mean however that q_S is not related to changes in E. This relationship is summarized by the following claim:

Claim. Price q_S paid by banks for capital bought from i-producers increases in E (for K kept constant).

Appendix A.3 Discussion about the assumptions of the model in the main text

Let us come back to the model in which the distribution of productivity A_I is non-trivial.

Independent and identically distributed aggregate shock. This assumption is made for three reasons.

First, it is made to eliminate the influence of shocks' persistence on agents' decisions. In particular, if it is assumed that Z is Markovian then $\pi(Z_L)$ and $\pi(Z_H)$ would be replaced by $\pi(Z_L|Z_{-1})$ and $\pi(Z_H|Z_{-1})$, respectively where $\pi(\cdot|Z_{-1})$ is probability measure of current aggregate shock conditional on the previous realization of Z - Z_{-1}. Then it would imply that q_S (and by market clearing conditions k_F, too) that is implicitly defined by (13) depends not only on E but also on Z_{-1}. Hence it would be hard to isolate the influence of E on k_F from the impact of agents' expectations about the realization of Z (captured by $\pi(Z_L|Z_{-1})$ and $\pi(Z_H|Z_{-1})$) on banks' decision about k_F. Since the former is the key force in my analysis and it is a channel that is significantly affected by changes in the intermediaries' market structure then it was isolated from influence of any additional factors.

Second, if despite the assumption about i.i.d. shocks, the model is able to generate persistent changes in economic aggregates then importance of the underlying acceleration mechanism (that works through the effect of E on k_F and Y in my model) is shown. A similar argument for using i.i.d. shocks is presented in Gertler (1989).
Third, this assumption enables me to calculate the closed-form solutions for the value function and the associated policies of producers (i.e., functions presented in Proposition 2).

Non-degenerate distribution of productivity A_I. Observe that if a continuous density f (with support $\mathbb{R}_+$) characterizes the distribution of A_I then supply of capital $S(q_S, K)$ is an increasing and differentiable function of q_S with $S(0, K) > 0$. This implies that we do not need to make any additional assumptions about parameters (analogous to condition (A1)) to guarantee the existence of RCE. This in turn means that we do not impose any additional constraints on parameters that could constrain parametrization/calibrations of the model. Moreover, this assumption gives rise to an additional channel through which price q_S (and conditions of banks' balance sheets) affects the real economy (in particular, the aggregate investment). This channel changes the extensive margin of investment since q_S affects the investment decisions of i-producers. For instance, if q_S jumps then more i-entrepreneurs find their investment opportunities profitable and hence more producers sell their entire capital to finance their investment project. This mechanism is absent if we consider the model with equal investment opportunities (e.g., $\mathbb{P}(A_I = 1) = 1$).

C-producers and i-producers that switch their types over time. Similarly to Bigio and d'Avernas (2021) a random and i.i.d. assignment of producer types is used. The randomness reduces the state space: if it is relaxed then we would have to keep track of both capital held by i-producers and c-producers. Assumption about the i.i.d. structure of these shocks could be replaced by the Markovian setup in which distribution of entrepreneurs across the two types is stationary of the corresponding Markov chain. This would make the notation more complex and worsen the clarity of exposition. Since replacing the assumption about i.i.d. assignments by Markovian ones would keep the qualitative features of my results unaffected then the simpler stochastic structure is followed in this work.

Different production technologies. Observe that there are two production technologies: a linear one (given by formula $A_I \cdot i$) and the Cobb-Douglas technology that is operated by c-entrepreneurs (that uses two inputs: capital and labor). This asymmetry is used (i.e., that investment goods are not produced by means of the Cobb-Douglas technology) to create a channel through which the amount of intermediation affects real economy. Observe that if investment goods are produced directly from consumption goods transferred by banks then this channel emerges in a natural way: the more capital k_F is transferred by banks from i-entrepreneurs to c-producers, the higher is the amount of resources (consumption goods) that can be used for production of new capital by i-producers. It is because q_S increases together with k_F and hence $q_S \cdot k_F$ grows as well.

Appendix A.4 Inefficiency of the RCE with competitive banks

Solution to the planner's problem. Let us compute the solution to the second stage problem (it is derived for given values of c_I, c_L, K' and K). Let us define the amount of resources available during the second stage:

$$\Omega\left(K, K', c_I, c_L\right) = A_C K^{\alpha} L^{1-\alpha} - \pi_I c_I - L c_L - K' + (1-\delta)K. \tag{A5}$$

Combining the FOCs associated with c_C and c_F yields:

$$c_C = Z c_F.$$

Plugging into the resource constraint yields:

$$c_F = \frac{\Omega\left(K, K', c_I, c_L\right)}{1 + Z\pi_C},$$

$$c_C = \frac{Z \cdot \Omega\left(K, K', c_I, c_L\right)}{1 + Z\pi_C}.$$

We use these results to reformulate the first stage problem:

$$P_1(K) = \max_{c_I, c_L, K'} \mathbb{E}_Z \left(\pi_C \cdot Z \cdot \log\left(\frac{Z \cdot \Omega\left(K, K', c_I, c_L\right)}{1 + Z\pi_C} \right) + \pi_I \log c_I \; + \right.$$
$$\left. + \log\left(\frac{\Omega\left(K, K', c_I, c_L\right)}{1 + Z\pi_C} \right) + L \log c_L + \beta P_1(K') \right).$$

Since we have log preferences we can extract terms $Z/(1{+}Z\pi_C)$ and $1/(1{+}Z\pi_C)$ which simplifies our further calculations. FOCs associated with per capita consumption levels c_I and c_L yield:

$$c_I = c_L = \frac{\Omega\left(K, K', c_I, c_L\right)}{1 + \mathbb{E}_Z Z \cdot \pi_C}.$$

First order condition for K' is:

$$\beta P_1^{'}(K') = \frac{1 + \mathbb{E}_Z Z \cdot \pi_C}{\Omega\left(K, K', c_I(K), c_L(K)\right)}. \tag{A6}$$

Let us plug formulas for c_I and c_L into (A5) and then combine it with (A6) to get:

$$\beta P_1^{'}(K') = \frac{1 + \mathbb{E}_Z Z \cdot \pi_C + L + \pi_I}{A_C K^{\alpha} L^{1-\alpha} - K' + (1-\delta)K}. \tag{A7}$$

Sources of inefficiency of the RCE allocation. Observe that producers cannot fully insure against the next period's value of idiosyncratic shock - they can use either "deposits" (if they are i-entrepreneurs) or purchase capital from intermediaries (if they are c-producers) but none of these options can insure them against being i-producer, insure them against becoming c-producer next period and simultaneously protect them against shifts in Z. Incompleteness of insurance markets faced by producers leads to a non-degenerate distribution of capital holdings and different consumption levels across entrepreneurs of the same type - this allocation feature is absent in case of the planner solution.
Incompleteness of insurance markets faced by intermediaries means that they cannot reduce the aggregate risk associated with shifts in demand for assets caused by changes in Z. Observe that if this risk is eliminated (e.g., by transfers that cover potential losses if the difference between the value of assets sold and deposits that has to be repaid is negative) then price q_S would move towards $q_B(Z_L)$. The latter price, by the previous discussion, does not depend on E and hence both the value of reallocated capital and aggregate investment becomes independent of history Z which establishes a qualitative similarity between the planner's solution and the RCE with transfers on the aggregate level. Hence the market incompleteness faced by banks induces them to reduce their intermediating activities which makes the reallocation of capital vulnerable to shifts in Z.

Appendix B Proofs for the model presented in the main text

Lemma 1. *Suppose that i and k_S solve (2). If $A_I > A_I^*(q_S)$ then $i > 0$ and $k_S = k$. If $A_I \leq A_I^*(q_S)$ then $i = 0$ and $0 < k_S < k$.*

Proof. Suppose that $A_I > A_I^*(q_S) = (1-\delta)/q_S$. By contradiction assume that optimal solution to (2) involves: $i \geq 0$ and $0 < k_S < k$. Consider the following deviation from the optimal plan: i-producers sells an additional portion of its capital κ ($0 < \kappa < k - k_s$) and spends a proportion $x = (1-\delta)/(A_I q_S)$ of the proceedings κq_S from this transaction on additional investment. Proportion $1 - x > 0$ (it is positive as $A_I > (1-\delta)/q_S$) is used for increasing consumption. The budget constraint is not violated. Observe that k'does not change:

$$\begin{aligned}\Delta k' &= A_I(i + x\kappa q_S) + (1-\delta)(k - k_S - \kappa) - A_I i - (1-\delta)(k - k_S)\\ &= A_I x\kappa q_S - (1-\delta)\kappa = A_I \frac{1-\delta}{A_I q_S}\kappa q_S - (1-\delta)\kappa = 0.\end{aligned}$$

At the same time c increased so this means that plan that involved $i \geq 0$ and $0 < k_S < k$ was not optimal.

Let us consider the case in which $A_I < A_I^*(q_S) = (1-\delta)/q_S$. Again, by contradiction suppose that optimal solution to (2) involves: $i > 0$ and $0 < k_S \leq k$. Consider the following deviation from the optimal plan: i-producer decreases investment by $0 < \iota < i$ and to guarantee that it budget constraint holds it decreases the amount of capital k that is sold (i.e., k_S) by $[Aq_S/(1-\delta)][\iota/q_S]$. At the same time he consumes the amount $[(1-\delta-Aq_S)/(1-\delta)]\iota$ of non-invested goods. As before, k' remains unaffected by this deviation:

$$\begin{aligned}\Delta k' &= A_I(i-\iota) + (1-\delta)(k + \frac{Aq_S}{1-\delta}\frac{\iota}{q_S} - k_S) - A_I i - (1-\delta)(k-k_S) \\ &= -A_I\iota + (1-\delta)\frac{Aq_S}{1-\delta}\frac{\iota}{q_S} = 0.\end{aligned}$$

At the same time, consumption increased so plan that involved $i > 0$ and $0 < k_S \leq k$ is not optimal.
Observe that i-producer remains indifferent between actions that either increase/decrease i and decrease/increase k_S when $A_I = A_I^*(q_S) = (1-\delta)/q_S$ so that WLOG we set $i = 0$ and $k_S = k$ in such situation. □

Proposition 2. *Decision rules and value function of an i-producer that has productivity level* $A_I < A_I^*(q_S)$ *are:* $c = [\phi/(1+\phi)]\omega_{I_0}$, $k^{'} = [1/(1+\phi)][(1-\delta)\omega_{I_0}/q_S]$, $V^{I_0} = \Psi^{I_0}(K,E) + (1+1/\phi)\log\omega_{I_0}$ *where* $\omega_{I_0} = q_S k$ *and* $\phi = (1-\beta)/[\beta\,(\Pi(A_L) + \Pi(A_H)\mathbb{E}Z)]$. *Decision rules and value function of an i-producer that has productivity level* $A_I \geq A_I^*(q_S)$ *are:* $c = [\phi/(1+\phi)]\omega_{I_P}$, $k^{'} = [1/(1+\phi)]A_I\omega_{I_P}$, $V^{I_P} = \Psi^{I_P}(K,E,A_I) + (1+1/\phi)\log\omega_{I_P}$ *where* $\omega_{I_P} = q_S k$. *Decision rules and value function of a c-producer are:* $c = [\phi Z/(1+\phi Z)]\omega_C$, $k^{'} = [1/(1+\phi Z)][(1-\delta)\omega_C/q_B]$, $V^C = \Psi^C(K,E,Z) + (Z+1/\phi)\log\omega_C$ *where* $\omega_C = (G(K)+q_B)\,k$.

Proof. Let us prove the case of the i-producer that has productivity level $A_I \geq A_I^*(q_S)$. The remaining cases are analogous and are omitted.
First, calculate i from the law of motion and plug it into the budget constraint. It follows that:

$$c + \frac{k'}{A_I} = q_S k.$$

Let us denote $\omega_{I_P} = q_S k$. This transforms our problem into a standard consumption-savings problem and enables me to use arguments presented by Alvarez and Stokey (1998) regarding dynamic programming problem with homogeneous objective function (in particular, solution to Bellman equation is unique).
To prove the exact functional forms of policies listed in Proposition 2, guess and verify method is used. Let us substitute the guesses of V^{I_P}, V^{I_0} V^C into i-producer's (that

has $A_I \geq A_I^*(q_S)$) Bellman equation:

$$V^{I_P}(k,K,E,A_I) = \max_{c>0,i\geq 0,k'>0} \Big\{ \log(c) + \beta \mathbb{E}_{Z,Z',A_I'} \left(\pi_I \cdot \mathbb{P}_{A_I}\left(A_I \geq A_I^*(q_S')\right) \times \right.$$
$$\times \left(\Psi^{I_P}(K',E',A_I') + \left(1+\frac{1}{\phi}\right)\log\omega'_{I_P}\right) +$$
$$+ \pi_I \cdot \mathbb{P}_{A_I}\left(A_I < A_I^*(q_S')\right) \cdot \left(\Psi^{I_0}(K',E') + \left(1+\frac{1}{\phi}\right)\log\omega'_{I_0}\right) +$$
$$\left. + \pi_C \cdot \left(\Psi^{C}(K',E',Z') + \left(Z'+\frac{1}{\phi}\right)\log\omega'_{C}\right) |K,E \right) \Big\}$$

subject to:

$$\begin{cases} c + \frac{k'}{A_I} = q_S k, \\ E' = E'(K,E,Z), \\ K' = K'(K,E), \end{cases}$$

By the fact that $\log\omega'_{I_P} = \log q'_s + \log k'$ (similarly for $\log\omega'_{I_0}$ and $\log\omega'_C$):

$$V^{I_P}(k,K,E,A_I) = \max_{c>0,k'} \log(c) + \frac{1}{\phi}\log k' + \bar{\Psi}^{I_P}(K,E)$$

subject to:

$$c + \frac{k'}{A_I} = \omega_{I_P}.$$

FOC is:

$$k' = \frac{1}{1+\phi} \cdot A_I \omega_{I_P}.$$

From the budget constraint we get:

$$c = \frac{\phi}{1+\phi}\omega_{I_P},$$

which confirms our guess for decision rules. Solutions for c and k' are plugged back to Bellman equation:

$$V^{I_P} = \Psi^{I_P}(K,E,A_I) + \left(1+\frac{1}{\phi}\right)\log\omega_{I_P}$$

which completes the proof. □

Proposition 3. *If A1 holds then decision rules and value function of intermediary are:* $c = (1-\beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K,E,Z) + [1/(1-\beta)]\log\omega_F$, $k_F = \Phi(K,E)\cdot e$, *where* $\omega_F = e + (q_B - q_S)k_F$.

Proof. The method used to prove Proposition 3 is analogous to one that was used to show that Proposition 2 holds. There is however one additional issue that needs to be solved: we need to show that the budget constraint

$$c + e' = e + (q_B - q_S)k_F$$

can be rearranged to the form of a constraint that is present in the standard consumption-savings problem. To prove that, let us first plug the guess for W_2 into W_1:

$$W_1(e, K, E) = \max_{k_F} \mathbb{E}_Z \left(\Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \omega_F \right),$$

substituting $\omega_F = e + (q_B(K, E, Z) - q_S(K, E))k_F$:

$$\begin{aligned} W_1(e, K, E) &= \\ &= \max_{k_F} \mathbb{E}_Z \left(\Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \left(e + (q_B(K, E, Z) - q_S(K, E))k_F \right) \right). \end{aligned}$$

The FOC under A1 reads:

$$\begin{aligned} \pi(Z_L) \cdot \frac{q_B(K, E, Z_L) - q_S(K, E)}{e + [q_B(K, E, Z_L) - q_S(K, E)]\, k_F} &+ \\ + \pi(Z_H) \cdot \frac{q_B(K, E, Z_H) - q_S(K, E)}{e + [q_B(K, E, Z_H) - q_S(K, E)]\, k_F} &= 0. \end{aligned} \quad \text{(B1)}$$

After a reformulation we get:

$$\begin{aligned} k_F &= \left(\frac{\pi(Z_H)}{q_S(K, E) - q_B(K, E, Z_L)} - \frac{\pi(Z_L)}{q_B(K, E, Z_H) - q_S(K, E)} \right) e = \\ &= \mathbb{E}_Z \left(\frac{\pi(Z)}{q_S(K, E) - q_B(K, E, Z)} \right) e \end{aligned}$$

which verifies my guess: $k_F = \Phi(K, E) \cdot e$. Let us show that k_F is positive. Observe that it is true iff:

$$\frac{\pi(Z_H)}{q_S(K, E) - q_B(K, E, Z_L)} > \frac{\pi(Z_L)}{q_B(K, E, Z_H) - q_S(K, E)},$$

which is equivalent to:

$$\pi(Z_H) \left(q_B(K, E, Z_H) - q_S(K, E) \right) + \pi(Z_L) \left(q_B(K, E, Z_L) - q_S(K, E) \right) > 0. \quad \text{(B2)}$$

It is shown later, that in equilibrium: $q_B(K, E, Z_H) - q_S(K, E) < 0$ and $q_B(K, E, Z_L) - q_S(K, E) > 0$. Additionally, the FOC (B1) can be written in the following form:

$$\mathcal{C}_1 \pi(Z_H) \left(q_B(K, E, Z_H) - q_S(K, E) \right) + \mathcal{C}_2 \pi(Z_L) \left(q_B(K, E, Z_L) - q_S(K, E) \right) = 0,$$

where $\mathcal{C}_1 > \mathcal{C}_2$ (because $q_B(K, E, Z_H) - q_S(K, E) < 0$
and $q_B(K, E, Z_L) - q_S(K, E) > 0$ in equilibrium). This implies that:

$$\pi(Z_H)\left(q_B(K, E, Z_H) - q_S(K, E)\right) + \frac{\mathcal{C}_2}{\mathcal{C}_1}\pi(Z_L)\left(q_B(K, E, Z_L) - q_S(K, E)\right) = 0,$$

where $\mathcal{C}_2/\mathcal{C}_1 < 1$. But this means that (B2) holds as the "weight" of 1 given to a positive term $q_B(K, E, Z_L) - q_S(K, E) > 0$ is higher than $\frac{\mathcal{C}_2}{\mathcal{C}_1}$ in the equation above. We are now in position to finish the proof in a standard way which was used for verification of policies and value functions of entrepreneurs. First note that since $k_F = \Phi(K, E) \cdot e$ then:

$$W_2(k_F, e, K, E, Z) = \tilde{W}_2(e, K, E, Z).$$

This means that:

$$\tilde{W}_2(e, K, E, Z) = \max_{c,e'}\left(\log c + \beta W_1(e', K', E')\right)$$

$$\text{subject to:}$$

$$\begin{cases} c + e' = \left(1 + (q_B - q_S)\Phi(K, E)\right)e = \omega_F, \\ E' = e', \\ K' = K'(K, E). \end{cases}$$

Guess for W_2 is plugged into W_1 and to the equation above:

$$\begin{aligned}\tilde{W}_2(e, K, E, Z) = \max_{c,e'}\Big(\log c + \beta\mathbb{E}_{Z'}\Big(\Psi^F(K', E', Z') &+ \\ + \frac{1}{1-\beta}\log\left(e' + (q_B - q_S)\Phi(K', E')e'\right)\Big)\Big)\end{aligned}$$

$$\text{subject to:}$$

$$\begin{cases} c + e' = \omega_B, \\ E' = e', \\ K' = K'(K, E). \end{cases}$$

This means that:

$$\tilde{W}_2(e, K, E, Z) = \max_{c,e'}\left(\log c + \frac{\beta}{1-\beta}\log e' + \tilde{\Psi}^F(K, E, Z)\right),$$
$$c + e' = \omega_F.$$

First order conditions are: $e' = \beta\omega_F$ and $c = (1 - \beta)\omega_F$. This confirms my guess for policy functions. We plug them back into Bellman equations to get:

$$\tilde{W}_2(e, K, E, Z) = \frac{1}{1-\beta}\log\omega_F + \Psi^F(K, E, Z),$$

but we know that $\omega_F = e + (q_B - q_S)k_F$ (i.e., ω_F is a function of k_F) so the initial formulation of W_2 is obtained:

$$W_2(k_F, e, K, E, Z) = \Psi^F(K, E, Z) + \frac{1}{1-\beta}\log \omega_F$$

and this completes the proof. □

Theorem 5. *If A1 holds then the solution to Equation* (13) *exists and is unique. Moreover, the aggregate reallocation of capital k_F increases with E. The following inequalities hold in equilibrium:*

$$\begin{aligned} D^{-1}(S(q_S), K, Z_H) &< q_S, \\ D^{-1}(S(q_S), K, Z_L) &> q_S. \end{aligned}$$

Proof. Let us first prove that:

$$\begin{aligned} D^{-1}(S(q_S), K, Z_H) &< q_S, \\ D^{-1}(S(q_S), K, Z_L) &> q_S. \end{aligned} \tag{B3}$$

It is a proof by contradiction: Suppose that in equilibrium:

$$\begin{aligned} D^{-1}(S(q_S), K, Z_H) &\geq q_S \\ D^{-1}(S(q_S), K, Z_L) &> q_S. \end{aligned}$$

This implies that (by the market clearing conditions in Definition 4): $q_B(Z_H) \geq q_S$ and $q_B(Z_L) > q_S$ (arguments K,E of q_B are omitted for clarity of exposition) but then banks have incentives to increase k_F which cannot happen in equilibrium.
Suppose that in equilibrium:

$$\begin{aligned} D^{-1}(S(q_S), K, Z_H) &< q_S \\ D^{-1}(S(q_S), K, Z_L) &\leq q_S. \end{aligned}$$

This implies: $q_B(Z_H) < q_S$ and $q_B(Z_L) \leq q_S$ but then banks have incentives to decrease k_F which cannot happen in equilibrium.
Suppose that in equilibrium:

$$\begin{aligned} D^{-1}(S(q_S), K, Z_H) &> q_S \\ D^{-1}(S(q_S), K, Z_L) &< q_S. \end{aligned}$$

This implies that $D^{-1}(S(q_S), K, Z_L) < D^{-1}(S(q_S), K, Z_H)$ and contradicts the fact that D is strictly decreasing in Z. Same argument excludes the possibility that:

$$\begin{aligned} D^{-1}(S(q_S), K, Z_H) &= q_S \\ D^{-1}(S(q_S), K, Z_L) &= q_S. \end{aligned}$$

This completes the proof of the auxiliary observation captured by inequalities (B3).Let us now turn to the main proof and prove existence first. The equilibrium condition (13) is reformulated to get:

$$\pi_H \frac{e + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)}{e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} = -(1 - \pi_H)\frac{D^{-1}(S(q_S), K, Z_L) - q_S}{D^{-1}(S(q_S), K, Z_H) - q_S}, \tag{B4}$$

where $\pi_L = \pi(Z_L)$ and $\pi_H = \pi(Z_H)$. The argument of S is omitted (i.e., argument K) to economize on notation. This reformulation was possible since by (B3) $D^{-1}(S(q_S), K, Z_H) - q_S \neq 0$ and by the log specification of preferences the non-zero consumption in problem (10) implies that $e+D^{-1}(S(q_S), K, Z_H)S(q_S)-q_S S(q_S) \neq 0$. By $\underline{q_S}$ denote q_S that satisfies:

$$D^{-1}(S(q_S), K, Z_H) - q_S = 0.$$

This number exists because there exists value q_S (because $D(q_S, K, Z_H)$ and $S(q_S)$ intersect only once - see Figure 4) such that: $S(q_S) = D(q_S, K, Z_H)$ (and this implies the existence of q_S that solves $D^{-1}(S(q_S), K, Z_H) = q_S$). Notice that for q_S converging to $\underline{q_S}$ from above, the LHS of the reformulated equilibrium condition (B4) is a finite positive number and the RHS converges to $+\infty$ (as the denominator is negative by (B3)).

Now let us define two additional numbers that are strictly greater than $\underline{q_S}$: the first one, $\bar{q}_{S,1}$solves:

$$e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S) = 0. \tag{B5}$$

There exists such a number greater than $\underline{q_S}$ because the LHS of (B5) evaluated at $\underline{q_S}$ is equal to $e > 0$. On the other hand since $\lim_{q_S \to +\infty} S(q_S) = \pi_I \cdot K$ and hence $\lim_{q_S \to +\infty} D^{-1}(S(q_S), K, Z_H)$ is a finite positive number, then the LHS of (B5) converges to $-\infty$ as $q_S \to +\infty$. This means that $\bar{q}_{S,1}$ exists by the Mean Value Property (since the LHS of (B5) is continuous). Observe that if q_S converges to $\bar{q}_{S,1}$ then the LHS approaches to $+\infty$ and the RHS is a finite positive number. The second one is: $\bar{q}_{S,2}$that solves:

$$D^{-1}(S(q_S), K, Z_L) - q_S = 0,$$

existence of which is guaranteed by identical reasons as those presented for $\underline{q_S}$ (observe that if the intersection of $D(q_S, K, Z_H)$ and $S(q_S)$ is well defined then the intersection of $D(q_S, K, Z_L)$ and $S(q_S)$ exists, too).

Let us consider two cases: $\bar{q}_{S,1} > \bar{q}_{S,2}$ and $\bar{q}_{S,2} \geq \bar{q}_{S,1}$. If $\bar{q}_{S,2} \geq \bar{q}_{S,1}$ then from what was said above the two continuous curves defined by the RHS and the LHS of the reformulated FOC (B4) must intersect at some point $q_S^* \in (\underline{q_S}, \bar{q}_{S,1})$ as one of them converges to $+\infty$ at one end of this interval while the other is positive (not necessarily strictly positive) and the situation is the other way round on the other end of the interval. If $\bar{q}_{S,1} > \bar{q}_{S,2}$ it can be observed that for q_S converging to $\underline{q_S}$ the RHS goes to $+\infty$ and the LHS is strictly positive. For q_S converging to $\bar{q}_{S,2}$ the RHS converges

Figure B1: Theorem 5 - existence, case $\bar{q}_{S,1} \leq \bar{q}_{S,2}$

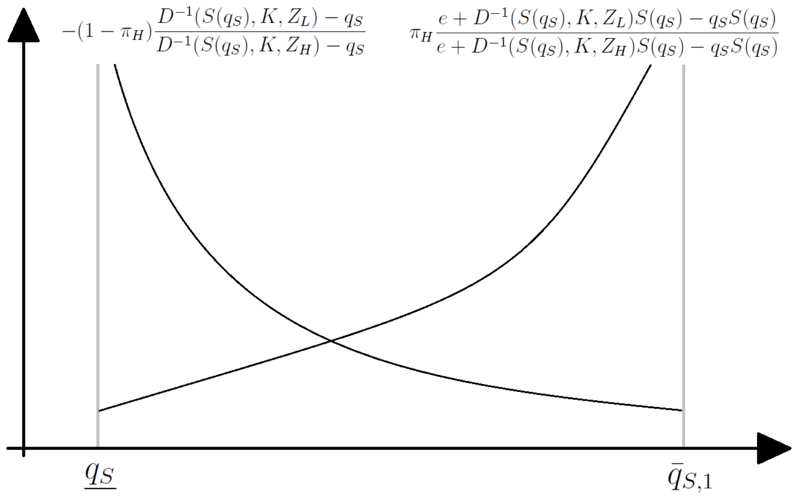
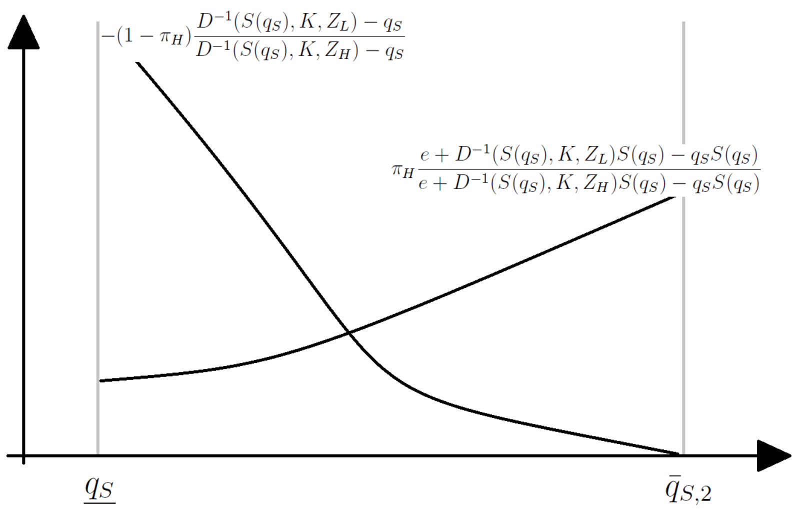


Figure B2: Theorem 5 - existence, case $\bar{q}_{S,1} > \bar{q}_{S,2}$



to 0 while the LHS approaches to a strictly positive number. Since the are both continuous for $(\underline{q_S}, \bar{q}_{S,2})$ then they must intersect at some point $q_S^* \in (\underline{q_S}, \bar{q}_{S,2})$. This means that a solution to (13) exists.
Let us prove uniqueness now. Another form of (13) is used:

$$(1-\pi_H)\frac{D^{-1}(S(q_S),K,Z_L)-q_S}{e+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)} = \\ = \pi_H \frac{q_S - D^{-1}(S(q_S),K,Z_H)}{e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}. \tag{B6}$$

Let us analyze the RHS of the reformulated FOC (B6) now. It can be calculated that:

$$\begin{aligned}
&\left(\frac{D^{-1}(S(q_S),K,Z_H)-q_S}{e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}\right)' = \\
&= \frac{1}{\left(e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)\right)^2}\cdot \\
&\quad \cdot\left\{\left(\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S),K,Z_H),Z_H)}-1\right)\times\right. \\
&\quad \times\left(e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)\right) + \\
&\quad - \Big[S'(q_S)\cdot(D^{-1}(S(q_S),K,Z_H)-q_S)+S(q_S)\times \\
&\quad \left.\times\left(\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S),K,Z_H),Z_H)}-1\right)\Big]\cdot\left(D^{-1}(S(q_S),K,Z_H)-q_S\right)\right\} = \\
&= \frac{1}{\left(e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)\right)^2}\cdot \\
&\quad \cdot\left\{\left(\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S),K,Z_H),Z_H)}-1\right)\cdot e + \right. \\
&\quad \left. - S'(q_S)\left(D^{-1}(S(q_S),K,Z_H)-q_S\right)^2\right\} < 0
\end{aligned} \tag{B7}$$

This is because $e > 0$, $S'(q_S) > 0$ and $D_{q_B} < 0$. This implies that the RHS is an increasing function of q_S. It is easy to see that analogous calculations prove that the LHS is a decreasing function of q_S.
This means that the RHS and the LHS of (B6) intersect at most once. But by our previous considerations we know that they do intersect so the point of the intersection is unique.Let us show that the aggregate reallocation of capital increases in E. From the bank's FOC and $E = e$ we get:

$$(1-\pi_H)\frac{D^{-1}(S(q_S),K,Z_L)-q_S}{E+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)}+ \\ +\pi_H\frac{D^{-1}(S(q_S),K,Z_H)-q_S}{E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}=0. \quad \text{(B8)}$$

Let us denote the LHS of (B8) by $B(q_s, E)$ (the second state variable – K – an be ignored as it is chosen in the first stage of the previous period and hence it remains unaffected by the choice of E in the second stage of the previous period) in From the proof of uniqueness we know that $B_{q_S}(q_s, E) < 0$. Let us check the sign of $B_E(q_s, E)$ now:

$$B_E(q_s,E)=-(1-\pi_H)\frac{D^{-1}(S(q_S),K,Z_L)-q_S}{\left(E+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)\right)^2}+ \\ -\pi_H\frac{D^{-1}(S(q_S),K,Z_H)-q_S}{\left(E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)\right)^2}. \quad \text{(B9)}$$

Since $B_E(q_s, E)$ is evaluated in equilibrium then bank's FOC must hold and then $\pi_H\frac{D^{-1}(S(q_S),K,Z_H)-q_S}{E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}$ is substituted for $(1-\pi_H)\frac{D^{-1}(S(q_S),K,Z_L)-q_S}{E+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)}$ in (B9) to get:

$$B_E(q_s,E)=\pi_H\frac{D^{-1}(S(q_S),K,Z_H)-q_S}{E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}\cdot \\ \cdot\left(\frac{1}{E+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)}+ \right. \\ \left. -\frac{1}{E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}\right).$$

Observe that since by (B3) $D^{-1}(S(q_S), K, Z_H) - q_S < 0$ and by the fact that:

$$E+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)>E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)$$

value $B_E(q_s, E)$ evaluated in equilibrium is positive. The Implicit Function Theorem is used to obtain:

$$k'_F(E)>0.$$

This completes the proof. □

Proposition 6. *If A1 holds then decision rules and value function of monopolistic intermediary are:* $c = (1-\beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K,E,Z) + [1/(1-\beta)]\log\omega_F$, $k_{F,j} = \tilde{\Phi}(K,E)\cdot e$, *where* $\omega_F = e + (q_B - q_S)\,k_{F,j}$.

Proof. It is sufficient to show that the FOC with respect to $k_{F,j}$ of the following expression:

$$W_1(e,K,E) = \max_{k_{F,j}} \mathbb{E}_Z \left(\Psi^F(K,E,Z) + \frac{1}{1-\beta} \log \left(e + \left(q_B \cdot \left(\frac{k_F}{k_{F,j}} \right)^{1-\frac{1}{\epsilon}} - q_S \right) k_{F,j} \right) \right),$$

defines an implicit, linear relationship between e and $k_{F,j}$ - the rest of the proof is done exactly in the same way as in proof of Proposition 3.
The FOC reads:

$$\pi(Z_L) \cdot \frac{\frac{1}{\epsilon} q_B(K,E,Z_L) \left(\frac{k_{F,j}}{k_F}\right)^{(1/\epsilon)-1} - q_S(K,E)}{e + \left[q_B(K,E,Z_L) \left(\frac{k_{F,j}}{k_F}\right)^{(1/\epsilon)-1} - q_S(K,E) \right] k_F} + \\ + \pi(Z_H) \cdot \frac{\frac{1}{\epsilon} q_B(K,E,Z_H) \left(\frac{k_{F,j}}{k_F}\right)^{(1/\epsilon)-1} - q_S(K,E)}{e + \left[q_B(K,E,Z_H) \left(\frac{k_{F,j}}{k_F}\right)^{(1/\epsilon)-1} - q_S(K,E) \right] k_F} = 0. \quad \text{(B10)}$$

Since the symmetric case is considered in which $e = E$ and rational agents recognize that their decisions are identical then they know that $k_{F,j} = k_F$ and hence the FOC is:

$$\pi(Z_L) \cdot \frac{(1/\epsilon) q_B(K,E,Z_L) - q_S(K,E)}{e + [q_B(K,E,Z_L) - q_S(K,E)] k_F} + \\ + \pi(Z_H) \cdot \frac{(1/\epsilon) q_B(K,E,Z_H) - q_S(K,E)}{e + [q_B(K,E,Z_H) - q_S(K,E)] k_F} = 0. \quad \text{(B11)}$$

Observe that (B11) implies that there exists a linear relationship between $k_{F,j}$ and e: $k_{F,j} = \tilde{\Phi}(K,E) \cdot e$. This in turn means that the budget constraint can be reformulated:

$$\omega_F = e + (q_B - q_S) \tilde{\Phi}(K,E) \cdot e$$

and hence the problem of the monopolistic intermediary becomes a standard consumption-savings problem. □

Theorem 7. *Under A1 solution to Equation* (20) *exists and is unique. Additionally, in the RCE with monopolistically competitive intermediaries aggregate reallocation of capital k_F increases with E.*

Proof. First, observe that analogously to the proof of theorem 5, marginal profit from intermediation in state Z_H:

$MP(Z_H)=\pi(Z_H)\cdot\{(1/\epsilon)q_B(K,E,Z_H)-q_S(K,E)\}\{e+[q_B(K,E,Z_H)-q_S(K,E)]\,k_F\}$ is negative and marginal profit from intermediation in state Z_L: $MP(Z_L)=\pi(Z_L)\cdot\{(1/\epsilon)q_B(K,E,Z_L)-q_S(K,E)\}\{e+[q_B(K,E,Z_L)-q_S(K,E)]\,k_F]\}$ is positive. If, by contradiction, $MP(Z_L) < 0 < MP(Z_H)$ then it violates the relationship:

$$\begin{aligned} q_B(K,E,Z_H) = D^{-1}\left(S(q_S(K,E)),K,Z_H\right) < D^{-1}\left(S(q_S(K,E)),K,Z_L\right) \\ = q_B(K,E,Z_L) \end{aligned} \tag{B12}$$

which is implied by $D_Z < 0$. If, by contradiction $MP(Z_L) > 0$ and $MP(Z_H) \geq 0$ or $MP(Z_L) < 0$ and $MP(Z_H) \leq 0$ then equality described by bank's FOC is violated. It is violated also for $MP(Z_L) = 0$ and $MP(Z_H) < 0$ and by $MP(Z_H) = 0$ and $MP(Z_L) > 0$. Observe that (B12) excludes the possibility that $MP(Z_H) = 0$ and $MP(Z_L) = 0$. This implies that if equilibrium exists then the following relationship must hold:

$$MP(Z_L) > 0 > MP(Z_H). \tag{B13}$$

Since logarithmic preferences imply: $e + [q_B(K,E,Z_H) - q_S(K,E)]\,k_F > 0$ and $e + [q_B(K,E,Z_L) - q_S(K,E)]\,k_F > 0$ then (B13) implies:

$$\frac{1}{\epsilon}q_B(K,E,Z_L) - q_S(K,E) > 0 > \frac{1}{\epsilon}q_B(K,E,Z_H) - q_S(K,E).$$

We are in position to prove existence of equilibrium. It can be done in an analogous way as in proof of existence of solution to (13), with the only difference that $\underline{q_S}$ is defined as q_s that satisfies:

$$S(q_S,K) = D(\epsilon q_S,K,Z_H)$$

and $\bar{q}_{S,2}$ is q_s that solves:

$$S(q_S,K) = D(\epsilon q_S,K,Z_L).$$

This means that solution to (20) exists.
Let us consider uniqueness now. Reformulated equilibrium condition (20) is:

$$\begin{aligned} (1-\pi_H)\frac{(1/\epsilon)D^{-1}(S(q_S),K,Z_L) - q_S}{e + D^{-1}(S(q_S),K,Z_L)S(q_S) - q_S S(q_S)} = \\ = \pi_H\frac{q_S - (1/\epsilon)D^{-1}(S(q_S),K,Z_H)}{e + D^{-1}(S(q_S),K,Z_H)S(q_S) - q_S S(q_S)}. \end{aligned} \tag{B14}$$

Let us calculate:

$$\left(\frac{(1/\epsilon)D^{-1}(S(q_S),K,Z_H)-q_S}{e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}\right)' =$$

$$= \frac{1}{\left(e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)\right)^2}\cdot$$

$$\cdot\left\{\left((1/\epsilon)\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S),K,Z_H),Z_H)}-1\right)\cdot e \ + \right.$$

$$- S'(q_S)\left(D^{-1}(S(q_S),K,Z_H)-q_S\right)^2 \ +$$

$$-(1-1/\epsilon)\cdot D^{-1}(S(q_S),K,Z_H)\cdot S(q_S) +$$

$$\left. +(1-1/\epsilon)\cdot q_S\cdot S(q_S)\cdot\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S),K,Z_H),Z_H)}\right\}<0.$$

It is because all terms in braces are negative (by the fact that $S' > 0$, $D_{q_B} < 0$, $D^{-1} > 0$, $S > 0$ and $\epsilon > 1$). This means that the LHS of (B14) decreases in q_S and the RHS increases in q_S. Since we know that they intersect (by existence) it means that solution to (B14) is unique. To prove that the second part of the statement in the theorem is true, it is shown that the partial derivative of the LHS of (20) with respect to e reads:

$$\pi_H\frac{(1/\epsilon)D(S(q_S),K,Z_H)-q_S}{E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}\cdot$$

$$\cdot\left(\frac{1}{E+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_S S(q_S)} \ + \right.$$

$$\left. -\frac{1}{E+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_S S(q_S)}\right)>0.$$

Since (by the proof of uniqueness) the partial derivative of the LHS of (20) with respect to q_S is negative. Hence by the Implicit Function Theorem $k'_F(E) > 0$. □

Proposition 8. *If the initial value of aggregate intermediaries' equity E and aggregate capital K are the same in both economies: the one with competitive banks and the one with monopolistically competitive intermediaries, then the amount of intermediated capital is strictly higher in economy with competitive banks than in economy with monopolistically competitive intermediaries.*

Proof. It suffices to investigate equilibrium conditions (13) and (20). Let us reformulate them to get:

$$(1-\pi_H)\frac{(1/\epsilon)D^{-1}(S(q_S),K,Z_L)-q_S}{e+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_SS(q_S)}= \\ =\pi_H\frac{q_S-(1/\epsilon)D^{-1}(S(q_S),K,Z_H)}{e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_SS(q_S)}, \quad \text{(B15)}$$

for economy with monopolistically competitive banks and:

$$(1-\pi_H)\frac{D^{-1}(S(q_S),K,Z_L)-q_S}{e+D^{-1}(S(q_S),K,Z_L)S(q_S)-q_SS(q_S)}= \\ =\pi_H\frac{q_S-D^{-1}(S(q_S),K,Z_H)}{e+D^{-1}(S(q_S),K,Z_H)S(q_S)-q_SS(q_S)}, \quad \text{(B16)}$$

for economy with competitive intermediaries. From proofs of Theorems 5 and 7 we know that the LHS of (B16) can be treated as decreasing function of q_S. On the other hand the RHS of (B16) increases in q_S. Analogous results hold for the RHS and the LHS of (B15). It is immediate that the curve defined by the LHS of (B15) is strictly below the curve defined by the LHS of (B16) since $\frac{1}{\epsilon}<1$. On the other hand the curve defined by the RHS of (B15) is strictly above the one defined by the RHS (B16). This implies that the point of intersection described by (B15) - q_S^{MC} is smaller than q_S^C that solves (B16). But this means that:

$$k_F^{MC}=S(q_S^{MC})<S(q_S^C)=k_F^C,$$

which completes the proof. □

Appendix C Proofs for the simplified model used for analyzing the long-run trade-off developed in Appendix A

Lemma 10. *Condition* (A2) *holds for all parameter values.*

Proof. Let us rewrite the condition that we want to prove:

$$\forall_K \frac{G(K)}{(1+\phi Z_H)(\pi_I/\pi_C)+\phi Z_H}>1-\delta. \quad \text{(C1)}$$

The strategy is the following: the upper bound for K is found (and it is denoted by $\tilde{K}$) in the dynamic model. Then the fact that (C1) holds for $\tilde{K}$ is shown. Then the fact that G decreases in K is used and hence the result for all K is obtained.

First, let us find $\tilde{K}$. Observe that the rate of aggregate investment satisfies:

$$I(q_S, K) = \frac{q_S \pi_I K}{1+\phi} < \frac{q_B(Z_L)\pi_I K}{1+\phi} = I(q_B(Z_L), K).$$

It is because in equilibrium $q_S < q_B(Z_L)$. It is clear (from (A3) and from (6)) that $q_B(Z_L)$ depends solely on one state variable, i.e. K so we do not need to keep track of E in the further considerations. Suppose that the economy experiences an infinitely long path of "good shocks" $Z = Z_L$. This means that (if we assume that K_0 is sufficiently small) under investment $I(q_B(Z_L), K)$ the aggregate capital K converges to steady state characterized by the following equation:

$$I(q_B(Z_L), K) = \delta K. \tag{C2}$$

This steady state is our candidate $\tilde{K}$. We calculate (the inverse demand function is used to replace $q_B(Z_L)$):

$$\begin{aligned} I(q_B(Z_L), K) &= \frac{q_B(Z_L)\pi_I K}{1+\phi} = \\ &= \frac{G(K)}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H} \cdot \frac{\pi_I K}{1+\phi} = \\ &= \frac{(1-\alpha)^{1/\alpha}\left(\frac{\alpha}{1-\alpha}\right) A_C^{1/\alpha}(1-\alpha)^{(\alpha-1)/\alpha} A_C^{(\alpha-1)/\alpha} (\pi_C K)^{\alpha-1} L^{1-\alpha}}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H} \cdot \\ &\cdot \frac{\pi_I K}{1+\phi}. \end{aligned}$$

We use (C2) to compute $\tilde{K}$:

$$\tilde{K} = \left[\frac{\alpha A_C \pi_I}{\delta\left((1+\phi Z_L)\frac{\pi_I}{\pi_C} + \phi Z_L\right)(1+\phi)}\right]^{1/(1-\alpha)} \frac{L}{\pi_C}.$$

Now it is shown that (C1) holds for $\tilde{K}$.

$$\frac{G(\tilde{K})}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H} > 1-\delta \iff \left[\frac{(1+\phi Z_L)\frac{\pi_I}{\pi_C} + \phi Z_L}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H}\right] \frac{1+\phi}{\pi_I} > \frac{1}{\delta} - 1$$

which is implied by our assumption about parameter values (A1). Since $\tilde{K}$ is an upper bound for all capital values then by the fact that G decreases with K we have:

$$\forall_K \; \frac{G(K)}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H} > 1-\delta$$

which completes the proof. □

Theorem 6. *If* (A1) *holds then solution to Equation* (A4) *exists and is unique.*

Proof. Let us rewrite the equilibrium condition (A4):

$$\pi(Z_L) \cdot \frac{D^{-1}(K, Z_L) - q_S}{E + [D^{-1}(K, Z_L) - q_S]\, \pi_I K} + \\ + \pi(Z_H) \cdot \frac{D^{-1}(K, Z_H) - q_S}{E + [D^{-1}(K, Z_H) - q_S]\, \pi_I K} = 0. \quad \text{(C3)}$$

It is clear that we need to consider values of q_S that satisfy: $q_S \in \left(D^{-1}(K, Z_H), D^{-1}(K, Z_L)\right)$ (by a similar reasoning to the one captured by the proof of Theorem 5). The LHS of (C3) is continuous for $q_S \in \left(D^{-1}(K, Z_H), \min\left\{D^{-1}(K, Z_L), \bar{q}_S\right\}\right)$ where $\bar{q}_S$ solves:

$$E + \left[D^{-1}(K, Z_H) - \bar{q}_S\right] \pi_I K = 0 \Longrightarrow \bar{q}_S = D^{-1}(K, Z_H) + \frac{E}{\pi_I K}.$$

For $q_S = D^{-1}(K, Z_H)$ the LHS of (C3) is positive. Suppose that $\min\left\{D^{-1}(K, Z_L), \bar{q}_S\right\} = D^{-1}(K, Z_L)$ then the LHS of (C3) is negative. If $\min\left\{D^{-1}(K, Z_L), \bar{q}_S\right\} = \bar{q}_S$ then the LHS of (C3) converges to $-\infty$ for $q_S \to \bar{q}_S$. This means that by the Mean Value Theorem, solution to (C3) exists.
Let us prove uniqueness now. Let us concentrate on the derivative of $(D^{-1}(K, Z) - q_S)/(E + \left[D^{-1}(K, Z) - q_S\right] \pi_I K)$ now:

$$\left(\frac{D^{-1}(K, Z) - q_S}{E + [D^{-1}(K, Z) - q_S]\, \pi_I K}\right)' = \frac{-E}{\left(E + [D^{-1}(K, Z) - q_S]\, \pi_I K\right)^2} < 0.$$

This means that the LHS of (C3) is strictly decreasing. This and existence of q_S that satisfies (C3) means that this solution is unique. □

Claim 6. *Price q_S paid by banks for capital bought from i-producers increases in E (for K kept constant).*

Proof. We will apply the Implicit Function Theorem to (A4). From the proof of Theorem Appendix C we know that the derivative of the LHS of (A4) decreases with q_S. Derivative of the LHS of (A4) with respect to E is:

$$-\left\{\pi_H \frac{D^{-1}(K, Z_H) - q_S}{E + [D^{-1}(K, Z_H) - q_S]\, \pi_I K} \cdot \left(\frac{1}{E + [D^{-1}(K, Z_H) - q_S]\, \pi_I K} + \\ - \frac{1}{E + [D^{-1}(K, Z_L) - q_S]\, \pi_I K}\right)\right\} < 0.$$

This implies that $\left(q_S(K, E)\right)'_E > 0$. □

Proposition 11. *The common lower bound on the supports of ergodic densities associated with E^C and E^{MC} is* 0.

Proof. Let us assume that upper bounds on densities' supports of E^C and E^{MC} exist (this will be shown in subsequent propositions). Let's denote them by $\bar{E}^C$and $\bar{E}^{MC}$. Take an arbitrarily small number $\mu > 0$. The idea of the proof (for the lower bounds $\bar{E}^C$and $\bar{E}^{MC}$) is to show that with some positive probability there exists a sufficiently long path of adverse shocks $\{Z_H, Z_H, ..., Z_H\}$ that the corresponding path of E_t^C (or WLOG the path of E_t^{MC}) decreases below μ. Then it is argued (by the Borel-Cantelli lemma) that for almost all trajectories $\{Z_t\}_{t=0}^{+\infty}$ there is an infinite number of such sequences $\{Z_H, Z_H, ..., Z_H\}$ and since the economy starts (i.e., when such sequence begins) from the lower level of E^C than $\bar{E}^C$ then the corresponding path of E_t^C will decrease below μ as well. Then by the fact that μ is arbitrary and that the number of these paths of $\{Z_H, Z_H, ..., Z_H\}$ is infinite we can argue that the value of density associated with the ergodic distribution of E^C is strictly positive for all positive numbers in the neighborhood of 0.
Let us consider the economy that starts at $\bar{K}^C$ and $\bar{E}^C$ in period 0. If it is affected by an adverse shock in this period then the next period's value of E is:

$$E_1^C = \beta \cdot \left(\bar{E}^C + \left(q_{B,1}\left(Z_H\right) - q_S\right) \cdot k_F\right) < \beta \bar{E}^C.$$

This inequality follows because for $Z = Z_H$ margin $q_{B,1} - q_S$ is negative in equilibrium. Using the same argument it is easy to see that:

$$E_t^C < \beta^t \bar{E}^C.$$

This means that there exists $t = T$ such that $E_T^C < \beta^T \bar{E}^C < \mu$ (because $\beta \in (0,1)$). This means that with probability $\left(\mathbb{P}(Z = Z_H)\right)^T > 0$ economy that starts $\bar{K}^C$ and $\bar{E}^C$ in period 0 has bank's equity lower than μ in period T. Now, by the Borel-Cantelli lemma we know that with probability 1 there is an infinite number of sequences $\{Z_H, Z_H, ..., Z_H\}$ of length T (within the sequence $\{Z_t\}_{t=0}^{+\infty}$) such that E^C falls below μ (at the end of the corresponding sequence of endogenous state variables) for an infinite number of times. This means that measure of the ergodic distribution of E^C that is accumulated in $(0, \mu)$ is positive. If the ergodic density exists then it means that it is positive for all positive numbers in a small neighborhood of 0. The same reasoning applies for the lower bound of ergodic density associated with E^{MC}. □

Proposition 12. *If* $\mathbb{P}\left(A_I = 1\right) = 1$ *and condition* (A1) *hold then the common lower bound on the supports of ergodic densities associated with K^C and K^{MC} is* $\underline{K} = \left(\frac{\Psi}{\delta}\right)^{1/(1-\alpha)}$ *where Ψ is a function of parameters.*

Proof. The strategy of the proof is the following. Let us first find an intuitive candidate $\underline{K}$ for the lower bound of the support of ergodic density of K^C (the proof for K^{MC} is the same). Then it is argued that there is a positive probability that the

economy experiences a sufficiently long path of "bad" shocks $\{Z_H, Z_H, ..., Z_H\}$ so that the aggregate capital in this economy falls below $\underline{K} + \eta$ where $\eta > 0$ is an arbitrarily small positive number. At the end the Borel-Cantelli lemma is used again to argue that the probability that $\{Z_H, Z_H, ..., Z_H\}$ occurs infinitely many times (within the sequence $\{Z_t\}_{t=0}^{+\infty}$) is 1 which implies that the measure of the ergodic distribution of K^C that is accumulated in $(\underline{K}, \underline{K} + \eta)$ is positive.
Let us first notice that the market clearing for "loans" in the economy in which $\mathbb{P}(A_I = 1) = 1$ is:

$$\pi_I K = \left[\frac{1}{1+\phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1+\phi Z} - 1\right] \cdot \pi_C \cdot K$$

which implies the following formula for q_B:

$$q_B(K, Z) = \frac{G(K)}{(1+\phi Z)\frac{\pi_I}{\pi_C} + \phi Z}. \tag{C4}$$

Additionally, notice that the formula for the aggregate output of new capital is:

$$I(q_S, K) = \frac{q_S(E, K)}{1+\phi}\pi_I K$$

which is implied by 3 and the fact that all i-entrepreneurs sell their entire stock of capital when condition (A1) holds. Now let us consider a hypothetical economy (which is signed by a subscript H) in which the aggregate output of new capital is:

$$I_H(K) = \frac{q_B(K, Z_H)}{1+\phi}\pi_I K.$$

Since in equilibrium $q_B(K, Z_H) < q_S(E, K)$ then $I_H(K) < I(q_S, K)$. Let us now derive a more tractable formula for $I_H(K)$:

$$\begin{aligned} I_H(K) &= \frac{q_B(K, Z_H)}{1+\phi}\pi_I K = \\ &= \frac{G(K)}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H}\frac{\pi_I K}{1+\phi} = \frac{\alpha A_C(\pi_C/L)^{\alpha-1}}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H}\frac{\pi_I}{1+\phi}K^\alpha = \Psi K^\alpha \end{aligned}$$

where Equation (C4), the formula for G are used and the following object is defined:

$$\Psi = \frac{\alpha A_C(\pi_C/L)^{\alpha-1}}{(1+\phi Z_H)\frac{\pi_I}{\pi_C} + \phi Z_H}\frac{\pi_I}{1+\phi}.$$

Now, it is easy to see that the hypothetical economy is deterministic and has two steady states: the one that is a trivial one with $K_{H,ss} = 0$ and the second with $K_{H,ss}$ that solves:

$$\Psi K^\alpha = \delta K.$$

This means that the non-trivial steady state satisfies $K_{H,ss} = (\Psi/\delta)^{1/(1-\alpha)}$. This value becomes a candidate for the lower bound $\underline{K}$.
Let us come back to the economy in which the output of new capital is $I(q_S, K)$. It will be shown that for an arbitrarily small positive number $\eta > 0$ there exists a finite number N such that for the realization $\{Z_H, Z_H, ..., Z_H\}$of length N the path of the economy's capital stock jumps into neighborhood $(K_{H,ss}, K_{H,ss} + \eta)$. The neighborhood $(K_{H,ss} - \eta, K_{H,ss})$ is ignored because if the economy drops into that region then it either converges to $K_{H,ss}$ in a monotone manner (in case of an infinite realization of Z_H which occurs with probability 0 or it jumps above $K_{H,ss}$ and never returns to $(K_{H,ss} - \eta, K_{H,ss})$. Both cases imply that ergodic measure of $(K_{H,ss} - \eta, K_{H,ss})$ is 0. Let us take two arbitrary, positive numbers η_1 and η_2 that satisfy:

$$\eta_1 + \eta_2 = \eta.$$

For $\eta_1 > 0$ let us construct a curve $I_H^{\eta_1}(K) = s(\eta_1) + I_H(K)$ such that $I_H^{\eta_1}(K)$ intersects with δK at $K_{H,ss} + \eta_1$. Suppose that $I_H^{\eta_1}(K)$ characterizes the investment rate in yet another hypothetical economy called economy η_1. It is obvious that since the aggregate amount of capital in economy η_1 converges to $K_{H,ss} + \eta_1$ then for there exists a finite number of periods N_1 during which economy η_1 that starts at $K \in [K_{H,ss}, \bar{K}^C]$ drops into $(K_{H,ss}, K_{H,ss} + \eta_1 + \eta_2)$.
Now, for each $K \in [K_{H,ss}, \bar{K}^C]$ let us define a number $\tilde{E}(K)$ for which $I\left(q_S\left(\tilde{E}(K), K\right), K\right) = I_H^{\eta_1}(K)$. This number exists by the continuity of q_S in E (which follows by the bank's FOC combined with equilibrium conditions) and by the fact that $\lim_{E\to 0} q_S(E, K) = q_B(K, Z_H)$. It is easy to see (again, by the bank's FOC combined with equilibrium conditions) that $\tilde{E}(K)$ is continuous. This means that it attains a minimum for $K \in [K_{H,ss}, \bar{K}^C]$ (a compact set). Let us denote it by K_{min} and by $N_2(K_{min})$ let us denote a natural number that satisfies (by the proof of Proposition 11):

$$\tilde{E}(K_{min}) > \beta^{N_2(K_{min})}\bar{E}.$$

This is clear that the output of new capital in economy that starts with any $K \in [K_{H,ss}, \bar{K}^C]$ and $\bar{E}$ falls below $I_H^{\eta_1}(K)$ if it experiences a sequence $\{Z_H, Z_H, ..., Z_H\}$ of length $N_2(K_{min})$. Since $I(q_S, K)$ remains below $I_H^{\eta_1}(K)$ if the sequence of "bad" shocks continues then it shrinks and it drops into the region $(K_{H,ss}, K_{H,ss} + \eta_1 + \eta_2)$ faster than the hypothetical economy η_1. This means that the "true" economy needs at most $N_1 + N_2(K_{min})$ (a finite number) of periods to find itself in $(K_{H,ss}, K_{H,ss} + \eta_1 + \eta_2)$. We set $N = N_1 + N_2(K_{min})$ and notice that $\pi(Z_H)^N$ is a strictly positive number. Now by the Borel-Cantelli lemma we know that the number of sequences $\{Z_H, Z_H, ..., Z_H\}$ of length N within $\{Z_t\}_{t=0}^{+\infty}$ is infinite with probability 1. Since $\eta > 0$ was an arbitrarily small positive number then we conclude that ergodic density of K is positive in a neighborhood $(K_{H,ss}, K_{H,ss} + \eta)$ of $\underline{K} = K_{H,ss}$. □

Proposition 13. *If $\mathbb{P}(A_I = 1) = 1$ and condition* (A1) *hold then $\frac{d\bar{K}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ is negative.*

Proof. Let us study the limits $\bar{K}^C$ and $\bar{E}^C$ to which the economy with competitive banks converges if the sequence of "good" shocks $\{Z_L, Z_L, ..., Z_L\}$ is infinite. From the law of motion for capital and from the equation characterizing investment in the simplified version of the model:

$$I(q_S, K) = \frac{q_S(E, K)}{1+\phi} \cdot \pi_I \cdot K$$

we get that in the limit:

$$\bar{q}_S^C = \frac{\delta(1+\phi)}{\pi_I} \tag{C5}$$

which means that $\bar{q}_S^C$ is a function of parameters. The market clearing condition for capital ("loans") implies:

$$\pi_I \bar{K}^C = \left[\frac{1}{1+\phi Z} \cdot \frac{G\left(\bar{K}^C\right)}{\bar{q}_B^C} + \frac{1}{1+\phi Z} - 1\right] \cdot \pi_C \cdot \bar{K}^C$$

which implies that:

$$\bar{q}_B^C(Z, \bar{K}^C) = \frac{G\left(\bar{K}^C\right)}{(1+\phi Z)\,\frac{\pi_I}{\pi_C} + \phi Z}. \tag{C6}$$

Let us denote $\kappa(Z) = (1+\phi Z)\,\frac{\pi_I}{\pi_C} + \phi Z$. Observe that $\bar{k}_F^C = \pi_I \bar{K}^C$ so we can rewrite the bank's FOC as:

$$\begin{aligned} 0 = \pi(Z_L)\left(\bar{q}_B^C(Z_L, \bar{K}^C) - \bar{q}_S^C\right) \cdot \left(\bar{E}^C + \left(\bar{q}_B^C(Z_H, \bar{K}^C) - \bar{q}_S^C\right)\pi_I \bar{K}^C\right) + \\ + \pi(Z_H)\left(\bar{q}_B^C(Z_H, \bar{K}^C) - \bar{q}_S^C\right) \cdot \left(\bar{E}^C + \left(\bar{q}_B^C(Z_L, \bar{K}^C) - \bar{q}_S^C\right)\pi_I \bar{K}^C\right). \end{aligned} \tag{C7}$$

The last equation that characterizes the economy is the law of motion for banks' equity that is derived from the bank's FOC:

$$\bar{E}^C = \beta\left[\bar{E}^C + \left(\bar{q}_B^C(Z_L, \bar{K}^C) - \bar{q}_S^C\right)\pi_I \bar{K}^C\right] \tag{C8}$$

If we plug (C6) and (C8) into (C7) then we can calculate the long-run value of capital:

$$\bar{K}^C = \frac{L}{\pi_C}\left[\frac{\left\{\pi(Z_L)\frac{\beta}{1-\beta}\frac{1}{\kappa(Z_L)} + \left(\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta}\right)\frac{1}{\kappa(Z_H)}\right\}\alpha A}{\frac{1}{1-\beta}\bar{q}_S^C}\right]^{1/(1-\alpha)}. \tag{C9}$$

Since $\bar{q}_S^C$ is a function of parameters then $\bar{K}^C$ is, too.

Observe that an analogous system of equations can be constructed for monopolistically competitive banks. Equation that corresponds to combination of (C7) and (C8) in the monopolistic regime is:

$$\begin{aligned} 0 = \pi(Z_L)\left(\frac{1}{\epsilon}\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) \cdot \left(\bar{E}^{MC} + \left(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}\right)\pi_I \bar{K}^{MC}\right) \\ + \pi(Z_H)\left(\frac{1}{\epsilon}\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) \cdot \left(\bar{E}^{MC} + \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}\right)\pi_I \bar{K}^{MC}\right). \end{aligned} \tag{C10}$$

Since we can use the ”monopolistic” equivalent of Equation (C6) to eliminate $\bar{q}_B^{MC}$ then it can be concluded that equation (C10) defines $\bar{K}^{MC}$ as an implicit function of ϵ (as (C10) becomes an equation with one endogenous variable). This fact is used together with the Implicit Function Theorem to check the sign of $\frac{d\bar{K}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ and $\bar{K}^{MC} = \bar{K}^C$.
Let us define $F(\bar{K}^{MC}, \epsilon)$ as the RHS of the equation above. It is calculated (after plugging $\bar{E}^C = \bar{E}^{MC}$ from (C8)):

$$
\begin{aligned}
F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) = {} & \frac{\beta\pi_L}{1-\beta} \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) + \\
& + \pi_L \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) + \\
& + \frac{\beta\pi_L}{1-\beta} \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) + \\
& + \pi_L \cdot \bar{q}_{B,K}^{MC}(Z_H, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) + \\
& + \frac{\pi_H \cdot \bar{q}_{B,K}^{MC}(Z_H, \bar{K}^{MC})}{1-\beta} \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}\right) + \\
& + \frac{\pi_H \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC})}{1-\beta} \cdot \left(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}\right),
\end{aligned}
$$

observe that by (C6) $\bar{q}_B^{MC}$ is a function of $\bar{K}^{MC}$ and hence $\bar{q}_{B,K}^{MC}$ denotes the derivative with respect to $\bar{K}^{MC}$. Note that $\bar{q}_S^{MC} = \bar{q}_S^C$. We use (C6), formula for $G(\cdot)$ and the definition of $\kappa(Z)$ to obtain:

$$
\begin{aligned}
F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) = G'\left(\bar{K}^{MC}\right) \cdot \Bigg\{ & \left(\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L)\right) \times \\
& \times \cdot \left[\frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_L)} + \frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_H)}\right] + \\
& + \frac{2\pi(Z_L)\beta}{1-\beta} \cdot \frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_L)} \Bigg\}
\end{aligned}
$$

where the fact that $\bar{q}_{B,K}^{MC} = G'\left(\bar{K}^{MC}\right) \frac{1}{\kappa(Z)}$ is used (see Equation (C6)). Observe that from (C7) and (C8) we get:

$$
\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C = -\left(\frac{1-\beta}{\beta} + \frac{1-\pi(Z_L)}{\beta\pi(Z_L)}\right)\left[\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C\right]. \quad \text{(C11)}
$$

This relationship implies that:

$$
\begin{aligned}
\left(\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L)\right) \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} + \frac{2\pi(Z_L)\beta}{1-\beta} \cdot \frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} = \\
= -\left[\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L)\right] \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} > 0
\end{aligned}
$$

because we know that in equilibrium $\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC} > 0$. Plugging back to the expression for $F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1)$ yields:

$$F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) = G'\left(\bar{K}^{MC}\right) \cdot \left(\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L)\right) \times \\ \times \left[\frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_H)} - \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)}\right]$$

It is clear that since $G'\left(\bar{K}^{MC}\right) < 0$, $[\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}]/\kappa(Z_H) > 0$, $[\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}]/\kappa(Z_L) < 0$ then $F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) < 0$. Let us consider $F_\epsilon(\bar{K}^{MC}, \epsilon = 1)$ now:

$$F_\epsilon(\bar{K}^{MC}, \epsilon = 1) = -\frac{1}{\epsilon^2} \cdot \left\{\frac{\pi(Z_L)\beta}{1-\beta}\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C\right) + \right. \\ + \pi(Z_L)\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C\right) + \\ \left. + \frac{\pi(Z_H)}{1-\beta}\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C\right)\right\}.$$

Let us use (C11) again to calculate:

$$\frac{\beta}{1-\beta} \cdot \left(\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C\right) + \left(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C\right) = \\ = -\left(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C\right)\left[\frac{\pi(Z_H)}{\pi(Z_L)\left(1-\beta\right)}\right] > 0.$$

Plugging back to the formula for $F_\epsilon(\bar{K}^{MC}, \epsilon = 1)$ gives us:

$$F_\epsilon(\bar{K}^{MC}, \epsilon = 1) = \frac{1}{\epsilon^2}\frac{\pi(Z_H)}{1-\beta}G\left(\bar{K}^{MC}\right) \times \\ \times \left[-\frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_H)} + \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)}\right]$$

where the formula (C6) was used. Since $[\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}]/\kappa(Z_H) > 0$, $[\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}]/\kappa(Z_L) < 0$ then $F_\epsilon(\bar{K}^{MC}, \epsilon = 1) < 0$. By the Implicit Function Theorem we get the following result:

$$\frac{d\bar{K}^{MC}}{d\epsilon} = -\frac{F_\epsilon(\bar{K}^{MC}, \epsilon = 1)}{F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1)} < 0.$$

This completes the proof. Observe that it has not been shown that there is an infinite number of trajectories that approach $\bar{K}^C$ for $\{Z_t\}_{t=1}^{+\infty}$. Analytic proof of this fact (like it was in the case for the lower bounds) is much harder to construct so a numerical

verification is used to show that the trajectory that corresponds to a sufficiently long path of $\{Z_L, Z_L, ..., Z_L\}$ converges to $\bar{K}^C$. Then by a similar argument (i.e., the Borel-Cantelli lemma) one can argue that the mass of the ergodic distribution in the neighborhood of $\bar{K}^C$ is positive.
It is useful, however, to compute a more precise expression for $\frac{d\bar{K}^{MC}}{d\epsilon}$ (it will be useful to prove next propositions):

$$\frac{d\bar{K}^{MC}}{d\epsilon} = -\frac{F_\epsilon(\bar{K}^{MC}, \epsilon = 1)}{F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1)} = -\frac{1}{\epsilon^2}\frac{\frac{\pi(Z_H)}{1-\beta}}{\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta}}\frac{1}{1-\alpha}\bar{K}^{MC}$$

□

Proposition 14. *If* $\pi(Z_L)\beta > \alpha$, $\mathbb{P}(A_I = 1) = 1$ *and condition* (A1) *hold then* $\frac{d\bar{E}^{MC}}{d\epsilon}$ *evaluated at* $\epsilon = 1$ *is positive.*

Proof. Let us observe that by (C8), the long run value of bank's equity can be rewritten as:

$$\bar{E}^C = \frac{\beta}{1-\beta}\left(\bar{q}_B^C(\bar{K}^C, Z_L) - \bar{q}_S\right)\pi_I\bar{K}^C$$

where $\bar{q}_S$ is a function of parameters. (C6) is used to reformulate the equation above:

$$\bar{E}^C = \frac{\beta\pi_I}{1-\beta}\left(\frac{\alpha A_C L^{1-\alpha}\pi_C^{\alpha-1}}{\kappa(Z_L)}\left(\bar{K}^C\right)^\alpha - \bar{q}_S\bar{K}^C\right).$$

This defines $\bar{E}^C$ as a strictly concave function of $\bar{K}^C$. This function attains its maximum at:

$$\bar{K}_E^C = \frac{L}{\pi_C}\left(\frac{\alpha^2 A_C}{\bar{q}_S\kappa(Z_L)}\right)^{1/(1-\alpha)}$$

and it decreases for $\bar{K}^C > \bar{K}_E^C$. This inequality holds in our case. It is because (from (C9)):

$$\bar{K}^C = \frac{L}{\pi_C}\left[\frac{\left\{\pi(Z_L)\frac{\beta}{1-\beta}\frac{1}{\kappa(Z_L)} + \left(\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta}\right)\frac{1}{\kappa(Z_H)}\right\}\alpha A}{\frac{1}{1-\beta}\bar{q}_S}\right]^{1/(1-\alpha)} >$$

$$> \frac{L}{\pi_C}\left(\frac{\alpha^2 A_C}{\bar{q}_S\kappa(Z_L)}\right)^{1/(1-\alpha)}$$

which is equivalent to:

$$\pi(Z_L)\beta + [\pi(Z_L)(1-\beta) + \pi(Z_H)]\frac{\kappa(Z_L)}{\kappa(Z_H)} > \alpha$$

and since by assumption $\pi(Z_L)\beta > \alpha$ then the inequality above follows. Let us use Proposition 13: if ϵ increases then $\bar{K}^{MC}$ drops. Since the value of $\bar{K}^{MC}$ that corresponds to $\epsilon = 1$ satisfies $\bar{K}^{MC} = \bar{K}^{C} > \bar{K}^{C}_{E}$ and since $\bar{E}^{C}$ is strictly concave in $\bar{K}^{MC}$ then if $\bar{K}^{MC}$ drops in response to growth in ϵ then $\bar{E}^{MC}$ grows. □